1. For each description, draw an example of a simple graph meeting it or explain why none can exist:
   a) G is planar, not complete, and \( \chi(G) = 4 \)
   b) G's degree list is \([2, 2, 3, 4, 4, 5]\)
   c) G's chromatic number is 4, G is non-planar, and G is connected
   d) G has 7 vertices, is non-planar, and does not contain \( K_5 \) as a subgraph.

2. Solve the recurrence: \( a_n = a_{n-1} + 2a_{n-2} \); \( a_0 = 1 \), \( a_1 = 0 \)

3. If \( \{f_n\} \) is the Fibonacci sequence
   \( f_1 = 1, f_2 = 1, f_3 = 2, \text{etc.} \)
   use induction to prove that:
   \[ \sum_{i=1}^{n} f_i = f_{n+2} - 1, \quad n = 1, 2, 3, \ldots \]

4. Show that the proposition \( \neg p \Rightarrow p \) is logically equivalent to \( p \).

5a) Use the Euclidean algorithm to compute \( \gcd(198, 35) \).
   b) Solve the linear congruence: \( 35x \equiv 16 \pmod{198} \)
   (Find a solution in the range \( 0 \leq x < 197 \), and use it to find all integer solutions.)

6. Suppose we have the following fruit supplies available:
   Apples, pears, oranges, kiwis = all unlimited
   6 pomegranites; and 5 bananas...
   How many distinguishable 7-piece fruit baskets are possible? Answer the same question if at least 2 pears must be used.
3. Is the graph below planar or non-planar? Explain.

Consider the hypotheses:

"It is not rainy and it is not cold."
"If it is not July, then it is rainy or cold."
"If it is July or August, then it is rainy."

Show, using appropriate rules of inference, that these hypotheses lead to the conclusion:
"It is rainy."

4. There are 12 balls in an urn: 6 red, 4 blue, 2 green. 5 balls are selected at random. Find the probability that:
   a) Both green balls are among those picked.
   b) An equal number of green and blue balls are picked.
   c) More red balls than green balls are picked.

5. Use inclusion/exclusion to verify the chromatic polynomial of a 5-circuit is \( P(Z_5, x) = (x-1) [(x-1)^4 - 1] \).