Testing the series $\sum_{k=1}^{\infty} a_k$ for convergence

**Definitions**
1. $\sum_{k=1}^{\infty} a_k$ converges if $\{S_n\}$, $S_n = \sum_{k=1}^{n} a_k$, has a limit.
2. $\sum_{k=1}^{\infty} a_k$ converges absolutely if $\sum_{k=1}^{\infty} |a_k|$ converges.
3. $\sum_{k=1}^{\infty} a_k$ converges conditionally if $\sum_{k=1}^{\infty} |a_k|$ diverges but $\sum_{k=1}^{\infty} a_k$ converges.

**Known Series**
1. **p-series** $\sum_{k=1}^{\infty} \frac{1}{k^p}$ converges if $p > 1$; diverges if $p \leq 1$.
2. **Geometric Series** $\sum_{k=M}^{\infty} r^k$ converges if and only if $|r| < 1$, with sum $s = \frac{r^M}{1-r}$.

### Deciding what Test to Use for $\sum a_k$

- **Q:** Does $\sum a_k$ converge absolutely, converge conditionally, or diverge?

- **series diverges**
  - NO: $\lim_{k \to \infty} a_k = 0$?
  - YES: Does $\sum |a_k|$ converge?

  - if $|a_k|$ involves exponentials and/or factorials use ratio test
  - if $|a_k|$ involves polynomial or constant-power growth only, use limit comparison test with appropriate p-series
  - if $|a_k|$ looks like a function that can be integrated, use integral test

- **series converges absolutely**

- **series diverges**
  - NO: Are all terms in series non-negative?
  - YES: Is $\sum a_k$ an alternating series with $1a_{k+1} < 1|a_k|$

### Convergence Tests for Positive Series $\sum a_k$

1. **Ratio Test**
   - $L = \lim_{k \to \infty} \frac{a_{k+1}}{a_k}$
   - $L < 1$ converges
   - $L > 1$ diverges
   - $L = 1$ inconclusive

2. **Comparison Test**
   - if $a_k \leq b_k$ and $\sum b_k$ converges, so does $\sum a_k$. If $a_k \geq b_k$ and $\sum b_k$ diverges, so does $\sum a_k$

3. **Limit Comparison Test**
   - if $\lim_{k \to \infty} \frac{a_k}{b_k} = 1$, then $\sum a_k$ and $\sum b_k$ have the same behavior.

4. **Integral Test**
   - if $f(x)$ positive, continuous, decreasing and $f(x) = a_k$ on $[M, \infty)$, then $\sum_{k=M}^{\infty} a_k$ has same behavior as $\int_{M}^{\infty} f(x) \, dx$