In this problem, we investigate the behavior of the improper integral
\[ \int_{2}^{\infty} \frac{1}{x(\ln x)^p} \, dx \]

1. First let's look at two special cases; define \( f(x) = \frac{1}{x(\ln x)^2} \) and \( g(x) = \frac{1}{x \sqrt{\ln x}} \).

For each of these two functions, find the area under the graph between \( x=2 \) and \( x=t \), where \( t \) is left as a variable. Then evaluate this expression for \( t = 10, 100, 10^6 \) and \( 10^{12} \) using evalf. Can you tell what the limiting behavior is as \( t \to \infty \)?

2. Use the limit command on the expressions you found in part 1 to determine whether the improper integrals of \( f(x) \) and \( g(x) \) converge or diverge. Check these answers by computing the improper integrals directly using the int command.

3. With paper & pencil, show below that \( \int_{2}^{\infty} \frac{1}{x(\ln x)^p} \, dx \) converges if and only if \( p > 1 \).