Points from group discussion

- Safety
- Probabilistic rules, cellular automata
- Fluid flow model, working with PDE: almost never in an outstanding solution
- Changing lanes
- Represent circle as rectangle
- Compare real world examples

If one wants to use a PDE model in a MCM paper, I would suggest that you explicitly present a discretization.

\[ \frac{\partial p_i(t)}{\partial t} = \frac{\partial}{\partial x} \left( \frac{N_i(t)}{l} \right) \]

The outstanding solutions used one or both of the following frameworks:

- Divide the traffic circle into boxes, and write down differential equations for how the number of cars in each box changes with time.
  - Sometimes the model was even cruder, and the traffic circle was just divided into boxes where one box was for each road coming in, and one box for the traffic circle itself.
  - This crude framework was only used as the "simple model" in the outstanding solution papers, used to get some analytical results.
- Simulator of a discrete cellular automaton model (deterministic or probabilistic)
**Probabilistic modeling** is often useful in problem solving
- Introduce random variables or random numbers into either the simple or complex model to handle *unpredictability* or *heterogeneity*
  - One basic useful concept for modeling the inflow of agents at a specified rate, but with irregular entrance. (10 cars/minute, for example).
    - **Poisson process:** It generates a sequence of times that have random intervals between them.
      
      ![Diagram of Poisson process](image)

      - The random variable that describes the time between successive cars is an *exponentially distributed random variable*.

      ![Exponential density function](image)

      How do you *simulate random variables*?
      - Just use the built-in simulators in MATLAB, etc.
      - **Inverse transform method**
        - Useful for simple random variables, in particular it works for exponential random variables:

      \[
      T = \frac{-1}{\lambda} \ln(U)
      \]

      - For more complicated random variables, look into the **Rejection Method**

**General themes for MCM problem solving**
- Outstanding solutions often feature a *combination of two modeling frameworks*
  - One is *simple and crude*, but captures some essential features of the question, and with this model, one can get some *analytical results* (equations and formulas relating input and output).
○ One is more detailed and realistic, and solved by a computational method.
○ A very strong way to build on a simple and complex model in an MCM paper is to use the simple model as a test case to "validate" the complex model.

• Always make explicit how you are quantitatively interpreting the question (i.e., what do you mean by "best")
○ Sometimes unify costs and benefits by defining some overall utility that combines both.