If $z_\infty$ is infinite:

Can show $R_0$ is infinite, and $R_\infty$ is infinite

for all $\varepsilon > 0$.

$z_\infty$ is optimal value to

$$\max x \ e^{T x}$$

s.t. $A x = b$

$$c^T x \leq z^* + \varepsilon$$

$x \geq 0$.

Since this is infinite, there exists a vector $d$ with $e^T d > 0$, $A^T d = 0$, $c^T d \leq 0$, $d \geq 0$.

Let $x$ be $\frac{e^T d}{e^T d}$ optimal in (P).

Then $x + \lambda d$ is feasible in (P) for any $\lambda \geq 0$.

Further $c^T (x + \lambda d) \leq c^T x = z^*$.

So we must have $c^T d = 0$, and $x + \lambda d$ is optimal in (P) for any $\lambda \geq 0$.

Thus, the optimal value to $\min \{e^T x : A x = b, c^T x \leq z^* + \varepsilon, x \geq 0\}$ is infinite, so the optimal value $R_\infty$ to the relaxation

$\min \{e^T x : A x = b, c^T x \leq z^* + \varepsilon, x \geq 0\}$ must also be infinite.