CALCULUS I
Spring, 2000

QUIZ 3

Directions. Please enter the final answer only in the appropriate box. You must show all work to receive credit.

1. What is the equation of the secant line to \( g(x) = \sin(x) \) between \( x = \pi \) and \( x = 3\pi/2 \)?

   \[
   \begin{align*}
   \sin(\pi) &= 0 \\
   \sin(2\pi) &= -1 \\
   \end{align*}
   \]

   \[
   \begin{align*}
   m &= \frac{0 - (-1)}{\pi - 2\pi} = \frac{1}{\pi} \\
   \end{align*}
   \]

   \[
   \begin{align*}
   y &= \frac{2}{\pi}(x - \pi) \\
   y &= -2(x + 2) \\
   \end{align*}
   \]

2. Sketch the graph of a function, \( f(x) \), for which the following hold:

   (a) \( f(x) \) has an infinite discontinuity at \( x = 3 \)

   (b) \( \lim_{x \to -3} f(x) = -1 \)

   (c) \( \lim_{x \to -1} f(x) = f(-1) \)

   (d) The function is not continuous at \( x = -1 \).

   (e) \( \lim_{x \to -\infty} f(x) = -2 \)
3. Find the following limits:

(a) \[ \lim_{x \to 2^+} \frac{x}{x - 2} \]

\[
\frac{2}{0} \overset{\text{+}}{\searrow} +\infty \quad \frac{\text{+}}{\text{+}} \quad +\infty
\]

(b) \[ \lim_{x \to 2^+} \frac{14}{x - 3} \]

\[
\frac{14}{2 - 3} = -14
\]

(c) \[ \lim_{x \to \infty} \frac{3x^3 + x^2 + 5}{-2x^3 - 3x - 2} \]

\[
\frac{3}{-2} \text{ / coeff. of } x^6 \quad \frac{-3}{2}
\]

4. Clearly explain why \( f(x) \) given below is not continuous at \( x = 0 \).

\[ f(x) = \begin{cases} 
  x + 2 & \text{if } x > 0; \\
  -x + 2 & \text{if } x < 0.
\end{cases} \]

\[ f(2) \text{ is not defined.} \]

EXTRA CREDIT: (3PTS, all or nothing) Sketch \( h(x) = \frac{x^2 - 1}{|x-1|} \) on the interval \([-1,3]\).
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QUIZ 3

Directions. Please enter the final answer only in the appropriate box. You must show all work to receive credit.

1. What is the equation of the secant line to \( g(x) = \cos(x) \) between \( x = \pi/2 \) and \( x = \pi \)?

Choose e.g. \((\pi/2, 0)\)

\[
\begin{align*}
g(\pi/2) &= 0 \\
g(\pi) &= -1 \\
\text{secant line} &= \frac{y - 0}{x - \pi/2} = \frac{y + 1}{x - \pi} \\
&= \frac{-1}{\pi/2} \\
&= -\frac{2}{\pi} \\
y &= -\frac{2}{\pi} (x - \pi/2) \\
&\text{or } y = -\frac{2}{\pi} x + 1
\end{align*}
\]

2. Sketch the graph of a function, \( f(x) \), for which the following hold:

(a) \( \lim_{x \to 2^+} f(x) = 1 \)

(b) \( \lim_{x \to 2^-} f(x) = -1 \)

(c) \( \lim_{x \to -1^+} f(x) = f(-1) \)

(d) The function is not continuous at \( x = -1 \).

(e) \( \lim_{x \to \infty} f(x) = 3 \)

For Example

\[
\begin{array}{c}
\text{(Graph of the function)}
\end{array}
\]

\[
\begin{array}{c}
\text{(Graph of the function)}
\end{array}
\]
3. Find the following limits:

(a) \[ \lim_{x \to 2^+} \frac{2}{x - 2} \]

"-2" \[ \Rightarrow \text{check signs} \]

\[ \Rightarrow \Theta2 \rightarrow -\infty \]

(b) \[ \lim_{x \to 2^+} \frac{5}{x - 3} \]

\[ \frac{5}{2 - 3} = -5 \]

(c) \[ \lim_{x \to \infty} \frac{\sqrt{x + 5}}{3x - 2} \]

Power in bottom bigger. Like \( \frac{\sqrt{x}}{x} \)

4. Clearly explain why \( f(x) \) given below is not continuous at the given point.

\[ f(x) = \frac{2}{(x - 2)^2} \text{ at } x = 2 \]

\[ f(2) \text{ is not defined.} \]

EXTRA CREDIT: (3PTS, all or nothing) Sketch \( h(x) = \frac{x^2 - 1}{|x - 1|} \) on the interval [-1, 3].

\[ x < 1 \quad h = \frac{x^2 - 1}{-(x - 1)} = -(x + 1) \]

\[ x > 1 \quad h = \frac{x^2 - 1}{x - 1} = x + 1 \]