TRUE/FALSE QUESTIONS

THE REAL NUMBERS
Chapter #4

(1) Every element in a field has a multiplicative inverse.
(2) In a field the additive inverse of 1 is 0.
(3) In a field every element has an additive inverse.
(4) A field can have two distinct additive identities.
(5) For elements of a field, $x, y, z$, the sum $x + y + z$ is defined by the field axioms.
(6) There exists a field containing exactly two elements.
(7) In a field containing at least two elements the multiplicative identity is greater than the additive identity.
(8) Let $x$ be an element in an ordered field with $x > 0$. Then $x^{-1} > 0$.
(9) Let the field $\mathcal{F}$ be ordered by the subset $\mathcal{F}^+$ and let $x, y \in \mathcal{F}$. If $x > y$ then $x - y \in \mathcal{F}^+$.
(10) Let $x, y, z$ be elements of a field. If $xy = xz$ then $y = z$.
(11) Every subset of an ordered field has a least upper bound in the field.
(12) Every non-empty subset of the positive integers contains a greatest lower bound.
(13) Let $S$ denote a non-empty subset of an ordered field $\mathcal{F}$ that has the least upper bound property. If $S$ has an upper bound in $\mathcal{F}$ then $S$ contains a least upper bound.
(14) Let $S$ denote a non-empty subset of an ordered field $\mathcal{F}$ that has the least upper bound property. If $S$ has a lower bound in $\mathcal{F}$ then $S$ has a greatest lower bound in $\mathcal{F}$.
(15) A non-empty set of real numbers that is bounded above contains a largest element.

(16) Let $x$ and $y$ denote elements of an ordered field. Then

$$|x - y| \geq |x| - |y|.$$  

(17) The notion of an absolute value function exists in an ordered field independently of whether or not the field has the least upper bound axiom.

(18) A set of real numbers can have more than one least upper bound.

(19) There exists a smallest positive real number.

(20) The sum of two integers is an integer.

(21) The product of two integers is an integer.

(22) The additive inverse of an integer is an integer.

(23) The multiplicative inverse of an integer is an integer.

(24) The set of integers is unbounded.

(25) Let $x$ and $y$ be elements of an ordered field. Then

$$|xy| = |x||y|.$$  

(26) Let $x$ and $y$ be elements of an ordered field. Then

$$|x + y| = |x| + |y|.$$  

(27) Let $x$ and $y$ be elements of an ordered field. If $x < y$, then

$$|x| < |y|.$$  

(28) The symbol $\infty$ as it is commonly used, represents a real number.

(29) The set of rational numbers with the algebraic operations they inherit from $\mathbb{R}$ constitute a field.

(30) The set of rational numbers with the algebraic operations they inherit from $\mathbb{R}$ constitute a field with the least upper bound axiom.

(31) The set of irrational numbers with the algebraic operations they inherit from $\mathbb{R}$ constitute a field.

(32) The number, 0, is an irrational number.
(33) The number, $\sqrt{2}$, is a rational number.

(34) Between every two distinct real numbers there exists a rational number.

(35) Between every two distinct real numbers there exists an irrational number.

(36) The product of two irrational numbers is irrational.

(37) The sum of two irrational numbers is irrational.

(38) The product of two rational numbers is rational.

(39) The sum of two rational numbers is rational.

(40) A sequence is a function.

(41) A sequence of real numbers that converges to a real number is a Cauchy sequence.

(42) A sequence of real numbers that converges to $-\infty$ is a Cauchy sequence.

(43) A Cauchy sequence of real numbers converges to a real number.

(44) If $\{x_n\}_{n=1}^\infty$ and $\{y_n\}_{n=1}^\infty$ are convergent sequences in $\mathbb{R}$, then so is the sequence $\{x_n + y_n\}_{n=1}^\infty$.

(45) The following equality is valid:
$$\sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^k = 2.$$

(46) If the sequence of partial sums of a sequence $\{x_n\}_{n=1}^\infty$ converge to a real number $S$ then
$$\sum_{n=1}^{\infty} x_n = S.$$

(47) If two sequences of real numbers converge to the same real number, then the two sequences are the same.

(48) The sum of two sequences in $\mathbb{R}$ is defined independently of the convergence properties of the two sequences.
(49) The product of two sequences in \( \mathbb{R} \) is defined independently of the convergence properties of the two sequences.

(50) If the sum of two sequences in \( \mathbb{R} \) converges then each of the two sequences converges.

(51) If \( \{x_n\}_{n=1}^{\infty} \) is a convergent sequence in \( \mathbb{R} \) then \( \lim_{n \to \infty} x_n = 0 \).

(52) The elements of a convergent sequence in \( \mathbb{R} \) constitute a bounded set.

(53) If the elements of an infinite sequence in \( \mathbb{R} \) constitute a bounded set, then the sequence converges.

(54) The notion of a Cauchy sequence exists in a ordered field independently of whether or not the field has the least upper bound axiom.

(55) Let \( \epsilon > 0 \) and

\[
S_n = \sum_{k=1}^{n} \epsilon, \quad \forall n \in \mathbb{Z}.
\]

Then the sequence \( \{S_n\}_{n=1}^{\infty} \) is bounded.

(56) A real-valued function defined on a subset of \( \mathbb{R} \) that consists of a finite number of points, is necessarily continuous.

(57) There exists a real-valued function defined on \( \mathbb{R} \) that is continuous at all of the irrational numbers and discontinuous at all of the rational numbers.

(58) The sum of two real-valued functions each defined and continuous on the same subset of \( \mathbb{R} \) is continuous.

(59) The product of two real-valued functions each defined and continuous on the same subset of \( \mathbb{R} \) is continuous.

(60) Let \( f \) and \( g \) denote real-valued functions each defined on \([0, 1]\). If \( f \) is continuous and the product function, \( fg \), is continuous, then so too is \( g \).

(61) Let \( f \) and \( g \) denote real-valued functions each defined on \([0, 1]\). If \( f \) is continuous and the sum function, \( f + g \), is continuous, then so too is \( g \).