1. (a) Text, Colley: p. 466, #18.
   (b) Illustrate the result of part (a) by a set-up similar to p.449, #22, where the geometry of the greenhouse is the same, but instead the temperature varies as $T(x, y, z) = -\frac{2}{3}(x^2 + y^2) + 3(z - 2)^2$, with $H = -k\nabla T$ and the value of $k$ is the same on the ground and on the surface of the greenhouse. **Note:** This shows that the temperature distribution is in *equilibrium*.

2. (a) Text: p.487, #16
   (b) Let $D$ be a region in $\mathbb{R}^3$ bounded by a closed surface $S$, with a scalar function $\phi$ of class $C^1$ on and within $S$. Establish the “gradient theorem”,
   $$\iiint_D \nabla \phi dV = \iint_S \phi dS$$
   in Cartesian coordinates, by applying the divergence theorem to each component.

3. (a) Reference, Amagio & Rubinfeld: p. 127, #1
   (b) Verify the results of part (a) for *elliptic cylinder coordinates*
   $$r = (a \cosh u_1 \cos u_2, a \sinh u_1 \sin u_2, z), \quad a = \text{positive constant},$$
   $$0 \leq u_1 < \infty, \quad 0 \leq u_2 < 2\pi, \quad -\infty < z < \infty,$$
   in the Reference on p.389.

4. (a) Determine the function $y(x)$ that minimizes the integral
   $$I(y) = \int_0^\pi (2y \sin x + (y')^2)dx,$$
   subject to the conditions $y(0) = 0$, $y(\pi) = 0$.
   (b) How does the minimum obtained in part (a) compare with the minimum for $I(y)$ that results when only subject to the condition $y(0) = 0$? Do it first using the natural boundary condition (6.39) in the reference, and also by a solution procedure without applying a natural boundary condition, say by a standard method of calculus.

(over)
5. (a) Reference: p.282, #3
(b) Compare with the minimum obtained when instead of a natural boundary condition at $x = 1$, $\dot{y}(1) = 0$. 