1. The Gateway Arch in St. Louis, MO has the approximate shape of the inverted catenary curve $C$:

$$y = \left(b - a \cosh \left(\frac{x}{a}\right)\right), \quad \left(b - a \right) \leq x \leq \left(b - a \right) \frac{2}{a}, \quad b > a > 0,$$

in a suitable coordinate system Assuming it has constant linear mass density $\rho_0$, find the total mass of the Arch and determine its center of mass, in terms of $a$, $b$, and $\rho_0$. Numerically evaluate the center of mass, given that $b - a = 630$ ft, the height of the Arch. Suggestion: You may need Maple to find the value of $a$ or $b$.

2. (a) Given two scalar fields $u(x,y)$ and $v(x,y)$ of class $C^1$ on an open set $X$ containing the circular disk $D : x^2 + y^2 \leq 1$. Define two vector fields $\mathbf{F}$ and $\mathbf{G}$ as follows

$$\mathbf{F} = u(x,y)\mathbf{i} + v(x,y)\mathbf{j}, \quad \mathbf{G} = \left(\frac{\partial v}{\partial x} - \frac{\partial v}{\partial y}\right)\mathbf{i} + \left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)\mathbf{j}.$$ 

Use Green's theorem to evaluate

$$\iint_D \mathbf{F} \cdot \mathbf{G} \, dx \, dy,$$

if it is known that on $\partial D$, $u(x,y) = y$ and $v(x,y) = 1$.

(b) Text, p.413, #20.

3. (a) Text: p.414, #22 
   (b) Text: p.414, #23.

4. (a) Text: p.484, #5 
   (b) Text: p.430, #24. Note: The surface here is an example of part (a).

5. The fluid flow which has the velocity field

$$\mathbf{V} = (x^2 + 2y^2 + 2z^2 - a^2) \mathbf{i} - xy \mathbf{j} - xz \mathbf{k}, \quad a \text{ const.,}$$

(over)
is called Hill's spherical vortex. Let \( S \) denote the hemisphere \( x^2 + y^2 + z^2 = a^2, \ z \geq 0 \), and let \( \mathbf{n} \) be the unit normal that points out of the sphere.
(a) Calculate the circulation
\[
\Gamma = \oint_C \mathbf{V} \cdot d\mathbf{s},
\]
where \( C \) represents the circular base of the hemisphere in the plane \( z = 0 \).
(b) Verify Stokes’ Theorem for this flow: show by direct calculation that
\[
\Gamma = \iint_S \mathbf{\omega} \cdot \mathbf{n} dS,
\]
where \( \mathbf{\omega} = \nabla \times \mathbf{V} \) is the vorticity. \textit{Hint}: For a possible parametrization, see example 9 on p.443.
(c) Replace \( S \) by a simpler surface with the same boundary \( C \).