Solve the following initial-boundary-value problem.

\[ u_t + u = u_{xx} \]

\[ u_x(0, t) = 0 = u_x(\pi, t) \quad \text{for} \quad t > 0 \]

\[ u(x, 0) = 1 + 3 \cos 2x \quad \text{for} \quad 0 < x < \pi. \]

Note: in the process of finding all the useful solutions, you do not have to show all your work for all the cases, provided you are sure you have found all the nonzero solutions.