INTRODUCTION TO DIFFERENTIAL EQUATIONS, TEST 2.5
Sections 1-4, Spring 1998

Section ______________________  Name ______________________

Instructions. You are allowed to use one 8 1/2 × 11 inch sheet of paper of notes. No
calculators, computers, books, or cellular phones are allowed. Do not discuss your work
with anyone else. In order to receive credit, your work must be clear and legible.

Solve the following initial-boundary-value problem.

\[ u_{xx} + 4u_{yy} = 0 \]

\[ u_x(0, y) = 0 = u_x(\pi, y) \quad \text{for} \quad 0 < y < \pi \]

\[ u(x, 0) = 0 \quad \text{for} \quad 0 < x < \pi \]

\[ u(x, \pi) = 1 + 3 \cos 2x \quad \text{for} \quad 0 < x < \pi. \]

Note: in the process of finding all the useful solutions, you do not have to show all your
work for all the cases, provided you are sure you have found all the nonzero solutions.

\[ u(x, y) = X(x)Y(y) \quad \Rightarrow \quad X''Y + 4XY'' = 0 \quad \Rightarrow \quad \frac{X''}{X} = -\frac{4Y''}{Y} = -\sigma \]

\[ X'' + \sigma X = 0 \quad \Rightarrow \quad X(x) = A \cos \lambda x + B \sin \lambda x \]

\[ Y'' - \sigma Y = 0 \quad \Rightarrow \quad Y(y) = c_1 e^{\frac{\sqrt{\sigma}}{2}y} + c_2 e^{-\frac{\sqrt{\sigma}}{2}y} \]

\[ X'(0) = 0 = X'(\pi) \quad \Rightarrow \quad \lambda = n \quad \Rightarrow \quad \sigma = n^2 \]

\[ Y(0) = 0 = Y(\pi) \quad \Rightarrow \quad Y_n(y) = \sinh \frac{ny}{2} \]

\[ \sigma = 0 \quad \Rightarrow \quad X(x) = Ax + B \quad \Rightarrow \quad X_0(x) = \text{constant} \]

\[ u(x, y) = a_0 y + \sum_{n=1}^{\infty} a_n \cos nx \sinh \frac{ny}{2} \]

\[ 1 + 3 \cos 2x = u(x, \pi) = a_0 \pi + \sum_{n=1}^{\infty} a_n \cos nx \sinh \frac{n\pi}{2} \]

\[ u(x, y) = \frac{2}{\pi} + \frac{3}{\sinh \frac{\pi y}{2}} \cos 2x \sinh y \]

\[ T_{1,0} = a_0 = \frac{1}{\pi} \]

\[ \sinh \frac{2\pi}{2} a_2 = 3 \]

all other a's zero.