1. Since there is a one-to-one relationship between real numbers and their cubes, the following equations in real variables $x$ and $y$ appear to be equivalent:

\[
\begin{align*}
(x + y)^{1/3} + (x - y)^{1/3} &= 1 \\
\{ (x + y)^{1/3} + (x - y)^{1/3} \}^3 &= 2x + 3(x^2 - y^2)^{1/3} = 1 \\
x^2 - y^2 &= \left( \frac{1 - 2x}{3} \right)^3 \\
y^2 &= x^2 - \left( \frac{1 - 2x}{3} \right)^3
\end{align*}
\]

Setting $x = -1$, we obtain $y = 0$ from the last equation. But these values of $x$ and $y$ do not satisfy the first equation. Why not?

2. The nonsymmetric condition $a = b + c$ leads to the symmetric result

\[
a^4 + b^4 + c^4 = 2a^2b^2 + 2b^2c^2 + 2c^2a^2.
\]

Does that mean that $a = b + c$ is equivalent to $b = c + a$ and $c = a + b$?

3. An $m \times n$ rectangle is partitioned into $mn$ equal squares. A diagonal of the rectangle passes through many of these squares. How many?

Generalize to an $m \times n \times p$ rectangular box.

4. I have accepted a job in downtown Albany. I have requested my employer to find me a home less than a mile from the office, so that I could walk to work every day. I also want my home to be located so that I can take a different route every day of my five-year contract. The contract calls for my working five days a week, fifty weeks a year.

Streets in the downtown area are on a grid of squares, each 1/16 mile on the side, so my home must be fewer than 16 blocks from work.

(a) At how many different sites could my home be located?

(b) What is the nearest I can live to my office and still have access to enough different routes?

(c) At which sites could I get the most number of routes? For how many years could I work before running out of routes, if my home is located at a site of maximum routes?

5. Suppose that circles of equal diameter are packed tightly in $n$ rows inside an equilateral triangle. (The figure below illustrates the case $n = 4$.) If $A$ is the area of the triangle and $A_n$ the total area occupied by the $n$ rows of circles, what happens to the ratio $\frac{A_n}{A}$ as $n$ becomes large? What is the corresponding result for a square? Can you generalize the idea to spheres packed in a cube?