Quiz#3
Tuesday October 9, 2001
Math 2010-Sections 6 & 7--TA: Amina

Name: Amina (eladda@rpi.edu)

Problem#1: Polar Coordinates.
Use polar coordinates to find the volume of the solid region bounded above by the hemisphere
\[ z = \sqrt{16 - x^2 - y^2} \]
and below by the circular region \( R \) given by \( x^2 + y^2 \leq 4 \)

Please See Example #3 - p937

Problem#2: Sketching a Vector Field
Sketch some vectors in the vector field given by \( F(x,y) = <2x, y> \)

Please see Example 2 - p987
Problem #3: Curl and Div

a. Find \( \text{curl} \mathbf{F} \) and \( \text{div} \mathbf{F} \) for the vector field given by \( \mathbf{F}(x,y,z) = <2xyz, x^2z, x^2y> \).

b. Evaluate \( \text{curl} \mathbf{F} \) and \( \text{div} \mathbf{F} \) at \((2,1,-1)\).

a. \[ \text{curl} \mathbf{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xyz & x^2z & x^2y \end{vmatrix} = \left( x^2 \hat{i} \right) \hat{k} + \left( 2xy - 2xy \right) \hat{j} + \left( \frac{2x}{x} \right) \hat{l} \]
   \[ \hat{k} \]
   \[ = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \]

b. \[ \text{div} \mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial}{\partial x} (2xyz) + \frac{\partial}{\partial y} (x^2z) + \frac{\partial}{\partial z} (x^2y) \]
   \[ = 2yz \]

b. \[ \text{curl} \mathbf{F}(2,1,-1) = 0 \]
   \[ \text{div} \mathbf{F}(2,1,-1) = 2(1)(-1) = -2 \]

**END!**