# 11. a. if \( E = \) fraction of carrying capacity that can be held in safety
\[ K = \text{carrying capacity} \]
\[ N_t = \text{population at time } t \]

\( EK \) would give the number of organisms in safety if the population were at its carrying capacity. Dividing by \( N_t \) to get \( \frac{EK}{N_t} \) corrects the equation to give the percentage of any given population able to find refuge.

b. The fraction of hosts that find refuge \( \left[ \frac{EK}{N_t} \right] \) add to the fraction not parasitized.

The fraction of hosts that escape being parasitized due to random contacts based on the Poisson distribution is \( e^{-ap_t} \). This must be multiplied by the fraction that have not found refuge \( \left( 1 - \frac{EK}{N_t} \right) \) to give the fraction of hosts not in refuge that were not probabilistically parasitized.

Therefore
\[
\begin{align*}
\text{Fraction in refuge} & \quad \text{Fraction not in hiding} \quad \text{Fraction not randomly selected} \\
\frac{EK}{N_t} & \quad \left( 1 - \frac{EK}{N_t} \right) e^{-ap_t} & \quad f(N_t, p_t)
\end{align*}
\]
N_{k+1} = 2N_k f(N_k, P_k)

Substitute the fraction of hosts not parasitized in for $f(N_k, P_k)$

N_{k+1} = 2N_k \left( \frac{EK}{N_k} + (1 - \frac{EK}{N_k}) e^{-ap_k} \right)

= \lambda \left(EK + N_k e^{-ap_k} - EK e^{-ap_k} \right)

Then substitute the correction of carrying capacity for $\lambda$, host reproduction rate $\lambda(N_k) = \exp \Gamma(1 - N_k / K)$

N_{k+1} = \left[ \exp \Gamma(1 - N_k / K) \right] (EK + N_k e^{-ap_k} - EK e^{-ap_k})

d. Using the equation for density of parasitized pg. 84

P_{k+1} = N_k (1 - e^{-ap_k})

where the $c$ factor has been removed, consider this equation with the assumption that some host will find refuge.

Instead of $N_t$ in the density equation, use the number of host that are not in refuge $(E_x K)$ $(E_x K)$ subtracted from $N_t$ should give the number of exposed host - which then are subject to chance $(1 - e^{-ap_k})$ so $(N_t - EK)(1 - e^{-ap_k})$
All of the impossible mating combinations are given a frequency of zero.

For $\frac{F}{Aa \times AA}$ and $\frac{M}{Aa \times aa}$, female $V$ mates with male $U$ or $W$ based on their relative prevalence. The relative prevalence of male $U$ versus male $W$ is $\frac{U}{U+W}$.

The relative prevalence of male $W$ versus male $U$ is $\frac{W}{U+W}$.

So female $V$ times the relative prevalence of $U$ or $W$ gives mating frequency for these two combinations:

$$V \left( \frac{U}{U+W} \right)$$ for $Aa \times AA$ and$$V \left( \frac{W}{U+W} \right)$$ for $Aa \times aa$.

For $\frac{F}{AA \times AA}$ and $\frac{M}{aa \times AA}$, instead of the frequency for these crosses to be the product of the prevalence of the parents, (WCL), this chart assumes that the males can fertilize many women. Therefore, the woman become the real limiting factor, and the frequency of the match is much more dependent on the female frequency.

For $U$ for the $Aa \times aa$, and $W$ for the $aa \times AA$. 

In $\frac{F}{M}$. 

<table>
<thead>
<tr>
<th></th>
<th>AA</th>
<th>Aa</th>
<th>aa</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA × aa</td>
<td>0</td>
<td>Aa</td>
<td>0</td>
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<td>Aa × AA</td>
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<td>AA × AA</td>
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<tr>
<td>aa × AA</td>
<td>0</td>
<td>Aa</td>
<td>0</td>
</tr>
<tr>
<td>(w)</td>
<td>0</td>
<td>(w)</td>
<td>0</td>
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</tbody>
</table>

C. Add each column of the table:

\[ U_{n+1} = \frac{\nu u_n}{2(u_n + w_n)} + 0 \]

\[ V_{n+1} = u_n + \frac{\nu u_n}{2(u_n + w_n)} + \frac{\nu w_n}{2(u_n + w_n)} + w_n \]

\[ = u_n + w_n + \frac{\nu (u_n + w_n)}{2(u_n + w_n)} \]

\[ V_{n+1} = u_n + w_n + \frac{1}{2} V_n \]

\[ W_{n+1} = \frac{\nu w_n}{2(u_n + w_n)} + 0 \]

\[ \frac{1}{2} \frac{u_{n+1}}{a_{n+1} + w_{n+1}} + u_{n+1} + \frac{1}{2} V_{n+1} + \frac{1}{2} V \frac{w_{n+1}}{a_{n+1} + w_{n+1}} = 1 \]

\[ \frac{1}{2} V_{n+1} + w_{n+1} + u_{n+1} + \frac{1}{2} V \frac{u_n + w}{a_n + w} = 1 \]

\[ V_{n+1} = 1 - \frac{1}{2} V_n \]

new equations without w

\[ U_{n+1} = \frac{U_n}{1 - V_n} \]
20. e. \[ U_{n+1} = \frac{1}{2} V_n \frac{U_n}{1 - V_n} \]

\[ V_{n+1} = -\frac{1}{2} V_n \]

**steady state**

\[ \overline{U} = \frac{1}{2} \overline{V_n} \frac{\overline{U}}{1 - \overline{V_n}} \]

\[ 1 - \overline{V_n} = \frac{1}{2} \overline{V_n} \]

\[ 1 = \frac{3}{2} \overline{V_n} \]

\[ \overline{V_n} = \frac{2}{3} \]

\((\overline{U}, \overline{V})_{\text{steady state}}\)

\[ \overline{U} = \frac{2/3 \overline{U}}{2(\overline{U} + \overline{V})} = \frac{\overline{U}}{3(\overline{U} + \overline{V})} \]

\[ 3 \overline{U}^2 + 3 \overline{U} \overline{V} - \overline{U} = 0 \]

\[ 3(\frac{2}{3} - \overline{U})^2 + 3(\overline{U} - \overline{V}) \overline{U} - (\frac{2}{3} - \overline{U}) = 0 \]

\[ (\frac{1}{3} - \overline{U}) \left[ 3(\frac{1}{3} - \overline{U}) + 3 \overline{A} - 1 \right] = 0 \]

\[ \begin{cases} \overline{U} = \frac{1}{3} & 1 - 3 \overline{A} + 3 \overline{U} - 1 = 0 \\ \overline{A} = \text{anything} \end{cases} \]

\[ \begin{cases} \overline{U} = \frac{1}{3} & 1 - 3 \overline{A} + 3 \overline{U} - 1 = 0 \\ \overline{A} = \text{anything} \end{cases} \]

\[ \overline{U} + \overline{V} + \overline{W} = 1 \]

\[ \begin{cases} \frac{1}{3} + \frac{2}{3} + \overline{W} = 1 & \overline{W} = 0 \\ \overline{W} = \frac{1}{3} & \overline{U} = 0 \end{cases} \]

There does not seem to be a unique value for the \((\overline{U}, \overline{V})_{\text{steady state}}\).
stability

\[ u_{n+1} = \frac{1}{2} \left( \frac{2}{3} \right) \frac{u_n}{\frac{1}{3}} \]

This shows that \( E_{01} \) is at a non-specific steady state, and thus is stable. The same is assumed for \( \omega_n \).

\[ f. \quad \frac{u_{n+1}}{w_{n+1}} = \frac{\frac{1}{2} u_{n+1}}{\frac{1}{2} u_n} = \frac{u_n}{w_n} \]

With each generation, the ratio does not change.