Improving the Recovery of Interdependent Social Infrastructure Systems after an Extreme Event: Model and Application

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Abstract

This paper presents research pertaining to the restoration and recovery of critical social infrastructure systems after an extreme event. A social infrastructure system can be defined as a system that provides a service to a community and primarily relies on people to operate. This differs from a civil infrastructure system because of the reliance on physical components. All of these systems are highly interdependent and can be vulnerable to the hazards of an extreme event. This paper presents a mixed-integer programming (MIP) model designed to assist in post-disaster restoration decisions in order to maximize the performance of the social infrastructure systems in a community. This model, called the Civil Restoration with Interdependent Social Infrastructure Systems (CRISIS) model, is an extension of previous research conducted by Cavdaroglu et al. in solving the integrated interdependent network design and scheduling (IINDS) problem. The paper also presents a case study using a robust, artificial infrastructure dataset to test the model.

Keywords: Emergency Management; Mixed-Integer Programming; Infrastructure Systems; Networks

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1. Introduction

Infrastructure systems are crucial for the operation and wellbeing of a community, both during normal circumstances, but especially in a disaster situation (Cavdaroglu 2012). A disaster can be defined as an extreme event or series of events that disrupts the normal operations of a community and requires action in order to return those operations to normal. During a disaster, “although it may be the hardware (i.e., the highways, pipes, transmission lines, communication satellites, and network servers) that is the initial focus of discussions of infrastructure, it is actually the services that these systems provide that are of real value to the public.” (Little 2002) These services are needed to perform almost every type of societal action. Critical services such as power and potable water are critical for life in many circumstances, a service such as telecommunications is needed for the distribution of information, and the transportation network is required for the movement of goods and people.

Critical Infrastructure Systems can be divided into two categories, civil and social. Civil infrastructure systems are primarily physical systems and are designed as networks with services flowing from supply points, through transshipment points, and ultimately to demand points. These systems require relatively little human involvement; the role of individuals is primarily for the maintenance and control of the system and to repair damaged components when needed. Social infrastructure systems are also crucial for the welfare of a community. Social infrastructure systems are defined as systems which depend on human operators for the delivery of services and utilize physical components or structures to support their tasks. Some examples of social infrastructure systems are EMS, police, education, healthcare, personal banking, firefighting, and the fuel distribution supply chain.

Both civil and social infrastructure systems are quite vulnerable to damage and disruption as a result of an extreme event. This is a result of both the direct physical damage caused by a disaster as well as the cascading effects of that damage throughout each system. While there is some redundancy built into many infrastructure systems, even a slight amount of damage could lead to a disruption of services. This effect is compounded due to an important factor which is the interdependencies that exist between infrastructure systems.

“Our national and economic security rest upon a foundation of highly interdependent critical infrastructures.” (Rinaldi 2004)

Interdependencies are the relationships between components of different systems where a component of one system relies on the services from another system. This concept is one of the most important considerations when dealing with the restoration of civil infrastructure systems during a disaster (Cavdaroglu et al. 2013). Figure 1 below illustrates an example where a power substation is being flooded, which eliminates power services to a hospital. Most hospitals have backup power supplies;
therefore that hospital can still operate its own electronic equipment. A problem arises when the water pump that provides potable water to the hospital is on the same power distribution line as the hospital and cannot provide services to the hospital when the power supply is unavailable. Therefore, as a result of damage to the power infrastructure, the hospital is still receiving power service due to its generator, but is no longer receiving its potable water service.

One important factor of a community’s resilience is their ability to quickly and effectively recover from the effects of a disaster. The restoration of infrastructure systems is a large part of that recovery. The goal of the restoration process of any infrastructure system is to return their customers back to normal service levels as quickly as possible within the constraints of the resources that are available to them (Coffrin et al. 2012). As a result of the interdependent nature of these systems, it is important to consider how a restoration decision in one system affects the others.

The purpose of this research is to develop mathematical modeling formulations and optimization techniques to address the problem of decision making during an extreme event while accounting for the interdependencies among civil and social infrastructure systems. The goal is to provide insights into how the restoration of the civil infrastructure systems should be conducted to optimize the overall performance of the social infrastructure systems and thus, the resilience of the community. The reason why the performance of the social infrastructure systems can be directly related to resilience is because it is the social infrastructure systems that people rely on in their everyday lives for their occupations, for their health and wellbeing, and to perform basic daily functions. While civil infrastructures provide the means

**Fig. 1** Power-Water-Hospital Interdependency Example (Loggins and Wallace 2015)
to do these activities, the social infrastructures are how these are fulfilled. Secondly, having robust social infrastructure systems contributes to the resilience of a community during an extreme event. For example, having experienced and skilled emergency response organizations, which are prepared to handle an extreme event, results in fewer casualties or significant injuries. For these reasons, during an extreme event, is it vitally important for the social infrastructures to perform at their highest level and therefore, should have significant consideration in the decision making process.

Throughout this research, each of the infrastructure systems is treated as networks of nodes and arcs. Nodes can be classified as demand points, supply points, or transshipment points. The arcs are the mechanisms in which services flow between the nodes. A node does not necessarily exist in one system alone. A transshipment point in one infrastructure can be a demand point for another or a node can exist as a demand point for multiple infrastructure systems; this is how interdependencies are defined.

There are many different types of interdependencies. In a paper by Rinaldi et al. (2001), the authors describe four different types: physical, cyber, geographical, and logical. All of these interdependencies are modeled such that a change at one node could have an effect on its corresponding interdependent node. In many cases, especially physical interdependencies, if that node is not receiving the services that it needs, the node can no longer function in its system, in other cases, especially logical interdependencies, a negative consequence could be an increase in demand or a decrease in the amount of resources available. In any case, this would result in a negative impact on the performance of that system as a whole. One objective of the models produced from this research would be to reduce the negative impacts of all types of interdependencies.

Research into the field of critical infrastructure interdependencies was emphasized after the 9/11 World Trade Center Attack in 2001. That event is still being studied today because of the implications that infrastructure interdependencies had on the restoration process. Several case studies regarding that event were conducted and some were organized into a book published by the Natural Hazards Center in 2003. Included in this book was an article of lessons learned by O’Rourke et al. (2003) regarding the 9/11 disaster as well as a method for classifying and analyzing interdependencies by Wallace et al. (2003).

The consequence of infrastructure interdependencies during an extreme event is that it inevitably causes disruptions due to cascading failures. This was shown to be a problem by Van Eeten et al. (2011) who provides evidence of this issue from media and political sources. There are two types of cascading failures, within a single system or between multiple systems. Predicting what the disruptions will be following an extreme event requires a firm understanding of all of the systems involved. Dueñas-Osorio and Vemuru (2009) provide graph theoretic and optimization approaches for predicting cascading failures within a single power transmission network and determines based on network topology the resilience potential of the system (Winkler et al. 2010). In a series of papers by Reed et al. (2009; 2010a; 2010b),
the authors present a methodology for multiple infrastructure system inoperability, and discuss the cases of Hurricane Rita and Hurricane Katrina. Zimmerman and Restrepo (2009) quantifies interdependent system disruptions and provides examples from recent events.

Infrastructure restoration is one of the most difficult decision processes that can occur during a disaster. There are many considerations such as the amount of available resources, priorities of demand points, and the scheduling and timing of work tasks. There are many different methodologies on how restoration should take place and each infrastructure system is so diverse that special considerations need to be made for each. Orabi et al. (2009; 2010) provides a method for the restoration of transportation and more general networks which allocates resources using optimization techniques. Yan and Shih (2009) present a transportation restoration model integrated with emergency relief routing. The present paper expands upon research conducted initially by Lee et al. (2007; 2009) and then subsequently by Cavdaroglu et al. (2012; 2013) and Nurre et al. (2012; 2013) regarding the integrated restoration, assignment, and scheduling on multiple interdependent infrastructure systems. This work has been recently expanded by Coffrin et al. (2012) to include a more robust power flow model. For a recent overview of optimization models for infrastructure restoration in a single infrastructure system, we refer the reader to Nurre and Sharkey (2014).

2. Models of Critical Infrastructure Systems

Traditionally, modeling of critical infrastructure systems has concentrated on the civil systems that provide physical services to their customers. Social infrastructure systems are significantly different than civil infrastructure systems, therefore the way in which these systems are modeled also need to be different. These systems serve the community differently and also are affected by the hazards of an extreme event differently. This is due to multiple factors such as the level of human participation in these systems, the reliance on vulnerable physical components, and because demand for social infrastructure services can vary considerably more than the demand for civil infrastructure systems. Table 1 shows a summary of some of the relative differences between civil and social infrastructure systems from a modeling perspective. These differences exist specifically during an extreme event and were considered in the development of the models of social infrastructure systems.

<table>
<thead>
<tr>
<th>Civil Infrastructures</th>
<th>Social Infrastructures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physical networks, assume operation has little need for human interaction</td>
<td>Services are provided by individuals, supported by physical structures</td>
</tr>
<tr>
<td>Services are needed continuously</td>
<td>Services are requested on a discrete, as-needed basis</td>
</tr>
<tr>
<td>Services move from supply to demand</td>
<td>Services can either move from supply to demand (e.g., police) or move from supply to distribution points (e.g., fuel)</td>
</tr>
<tr>
<td>-----------------------------------</td>
<td>--------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Demand is more consistent and predictable</td>
<td>Demand can change over time</td>
</tr>
<tr>
<td>Damage is to the network itself and has no effect on demand for services</td>
<td>Damage is to the facilities that support the system and damage can have an effect on demand levels for services</td>
</tr>
<tr>
<td>A disruption in services at an interdependent node can result in customers not receiving services</td>
<td>A disruption can result in customers not receiving services, increases in demand for services, a decrease in resources, or an increase in the cost to provide services</td>
</tr>
</tbody>
</table>

The first step in the modeling process for infrastructure systems was to develop a classification scheme for these systems. This classification scheme groups the infrastructure systems by similar characteristics in order to generalize the way in which the models are developed for each system. All of the infrastructure systems that were considered in this research fit into four classification groups: civil infrastructure systems, emergency response network systems, distribution network systems, and systems of affiliated sets.

The first classification system, civil infrastructure systems, can be used to represent any system in which physical services flow from a supply point to a demand point. Examples of these systems include power, communications, water, and wastewater. The second classification, the emergency response network classification, can be generalized for systems in which there are services that flow between nodes from supply to demand, similar to civil infrastructure systems, but also consider the amount of time required to travel from supply to demand. Examples of these systems are fire, police, and EMS. The distribution network classification is for systems where services are flowing from supply points to distribution centers; customers then go to the distribution centers to obtain the services. Examples of these systems are the fuel distribution and personal banking infrastructures. In both of these cases, the supply and customers must travel through the transportation network to get to the distribution points. The locations of these distribution points, such as gas stations and ATMs, are assumed to be known and stationary.

The next classification is the affiliated set classification and this is for systems in which it is assumed that no flow exists between nodes; however an action, decision, or change at one node could have an effect on another. The healthcare infrastructure is an example of this type of system. If one hospital is shut down due to flooding or some kind of damage, then the demand for services at a neighboring hospital would increase. It is also possible for the nodes of the system to be independent
from one another where a change at one node has no impact on another. An example of this type of system would be parks and monuments. If a national monument is damaged, it would have no significant effect on other nodes in that system; however since there is still an impact on the community as a whole, it must still be considered in the decisions regarding the restoration of services. Table 2 below summarizes and provides examples of the different classifications of social infrastructure systems.

Table 2 Infrastructure Classification Scheme

<table>
<thead>
<tr>
<th>Classification</th>
<th>Description</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civil Infrastructure</td>
<td>A system in which goods or services flow in a network from supply to demand</td>
<td>Power, Water, Wastewater</td>
</tr>
<tr>
<td>Emergency Response</td>
<td>A system in which goods or services flow in a network from supply to demand</td>
<td>EMS, Police, Firefighters</td>
</tr>
<tr>
<td>Distribution Network</td>
<td>A system in which goods or services flow in a network from supply points to</td>
<td>Fuel distribution, Personal Banking,</td>
</tr>
<tr>
<td></td>
<td>distribution centers where they are obtained by customers</td>
<td>Food Distribution</td>
</tr>
<tr>
<td>Affiliated Set</td>
<td>A system in which an action or event at one node can have an effect on another, but there is no flow between them</td>
<td>Healthcare, Shelters, Government Facilities</td>
</tr>
</tbody>
</table>

Generalized models for all four classifications have been developed. In building these models, it was important to understand how the interdependencies between the systems affect the operations and decision making within each system. The models of civil infrastructure systems assumed that if component A depends on component B and component B is no longer functioning, then component A does not function. With regard to interdependencies among social infrastructure systems, which include the other three classifications, this may or may not be the case. For example, when component B loses function, component A could see an increase or decrease in its demand for services (e.g., if a nursing home in the healthcare system loses power, then the local EMS may see an increase in the number of calls from the nursing home until the power is restored). Understanding the different types of interdependencies that may exist in these social infrastructure systems was necessary in order to develop realistic models for these systems.

The following sections will discuss the modeling considerations for the three social infrastructure system classifications as well as for their integration with the restoration of civil infrastructure systems. The goal of this work is to determine how decisions made in both social and civil infrastructure systems affect the performance of those systems and the community as a whole. This model is called the Civil Restoration with Interdependent Social Infrastructure Systems (CRISIS) model. It is important to note
that this model is generalizable for any type of extreme event, whether a warning is given or not. Regardless of the type of event, it is necessary to understand what damage exists in each system and what time and resources are required to restore that damage. The damage assessment model presented in Loggins and Wallace (2015) is one method that can be used to understand the damage that is likely to occur given the characteristics of an extreme event. The CRISIS model assumes that both the damage as well as how long it takes to process each damaged component is known.

2.1 The CRISIS Model

The CRISIS model is designed to assist in decisions regarding the restoration of services following an extreme event. The objective of this model is to maximize the performance of the social infrastructure systems by restoring the civil infrastructure systems, while accounting for the interdependencies that exist among all of the systems. One important consideration is that while the objective of this model is to maximize the performance of the social infrastructure systems, it does not restore the social infrastructure systems, only the civil infrastructure systems and the physical components which support the social infrastructure systems. For example, in the police system, the model does not rebuild police stations that are damaged, but rebuilds the civil infrastructure components such as roadways on which police vehicles travel and power lines of which police stations receive power services. As a result of this restoration, the performance of the police system should improve. The CRISIS model is a mixed-integer programming model which contains elements of network flow, routing, scheduling, and assignment problems. The following sections outline the objective function and all of the constraint sets for the CRISIS model.

2.1.1 Civil Infrastructure Restoration

As stated previously, the objective of the CRISIS model is to maximize the performance of the social infrastructure systems by restoring the civil infrastructure systems while accounting for the interdependencies that exist among all of the systems. As a result, this model contains features of the scheduling and assignment of work crews for restoring civil infrastructure systems which support the social infrastructure systems. This section will describe work modified from previous research conducted by Cavdaroglu (2012) and Cavdaroglu et al. (2013) in solving the problem of restoring interdependent civil infrastructure systems, called the integrated interdependent network design and scheduling (IINDS) problem. This model is formulated as a mixed integer program and is an extension of another type of problem called the interdependent layered network (ILN) problem presented by Lee (2006) and Lee et al. (2007). The purpose of this model is to provide insights into the restoration decision for multiple interdependent infrastructure systems. The result of this model is the set of components that should be
repairs in each system, the assignment of those components to work crews, and the schedule in which the work should be completed for each work crew. The constraint set presented below is modified from the INDS model in order to account for multiple damage levels, the restoration of multiple systems, and the restoration of both nodes and arcs.

The network structure for a civil infrastructure system is defined by \( m \in M \) where \( m \) is a civil infrastructure system in the set of all systems \( M \). Let \( V^m \) be the set of all vertices in civil infrastructure system \( m \), \( V^m \) the set of partially damaged vertices, \( \bar{V}^m \) the set of fully damaged vertices, \( E^m \) the set of all edges, \( \bar{E}^m \) the set of partially damaged edges, and \( \bar{E}^m \) the set of fully damaged edges. Let \( V^{m+} \) be the set of supply nodes, \( V^{m-} \) the set of demand nodes, and \( V^{m=} \) the set of transshipment nodes for civil infrastructure system \( m \). Each arc can be defined by its starting and ending nodes or \((i, j) \in E^m\). Let \( k = 1, \ldots, K \) represent a single work crew in the set of available crews \( K \).

There are three sets of constraints associated with this portion of the model. The network flow constraints are the set of constraints that require infrastructure services flow from source to demand, arcs and nodes do not exceed their capacities, and that the flow through the arcs go in the correct direction. It is assumed that civil infrastructure systems provide their services instantaneously, therefore there is no time cost associated with the flow on each arc.

The scheduling constraints ensure that work crews are not working on more than one task during a time period, tasks are not duplicated by more than one work crew, and that once a task is started it is worked on until it is completely restored. In these constraints, \( a^m_{kijt} \) and \( a^m_{kit} \) is a decision variable representing whether work crew \( k \) is being utilized in time period \( t \) on arc \((i, j)\) or node \( i \) respectively, while \( p^{m}_{ij} \) and \( p^{m}_{i} \) are the processing times for arc \((i, j)\) or node \( i \) respectively. In these constraints, \( \beta^m_{ijt} \) is
and \( \beta_{ij}^m \) represent whether damaged arc \((i,j)\) or damaged node \(i\) respectively have been processed to completion.

\[
0 \leq x_{ij}^m \leq u_{ij}^m \beta_{ij}^m \quad \forall t = 1, ..., T, \forall m, \forall (i,j) \in E^m \cup \bar{E}^m
\]  
(8)

\[
\beta_{ij}^m - \beta_{ij}^m(t-1) = \sum_{k=1}^{K} \alpha_{kij}^m \quad \forall t = 1, ..., T, \forall m, \forall (i,j) \in E^m \cup \bar{E}^m
\]  
(9)

\[
\beta_{ij(t+1)}^m \geq \beta_{ij}^m \quad \forall t = 1, ..., T - 1, \forall m, \forall (i,j) \in E^m \cup \bar{E}^m
\]  
(10)

\[
\sum_{t=1}^{p_{ij}^m-1} \beta_{ij}^m = 0 \quad \forall t = 1, ..., T, \forall m, \forall (i,j) \in E^m \cup \bar{E}^m
\]  
(11)

\[
\sum_{k=1}^{K} \sum_{t=1}^{p_{ij}^m-1} \alpha_{kij}^m = 0 \quad \forall t = 1, ..., T, \forall m, \forall (i,j) \in E^m \cup \bar{E}^m
\]  
(12)

\[
\alpha_{kij}^m, \beta_{ij}^m \in \{0,1\} \quad \forall t = 1, ..., T, \forall m, \forall (i,j) \in E^m \cup \bar{E}^m, \forall k = 1, ..., K
\]  
(13)

\[
0 \leq \sum_{(j,\ell) \in E^m \cup \bar{E}^m} x_{j\ell}^m \lesssim u_{i\ell}^m \beta_{i\ell}^m \quad \forall t = 1, ..., T, \forall m, \forall i \in \bar{V}^m \cup \bar{\bar{V}}^m
\]  
(14)

\[
\beta_{it}^m - \beta_{it}^m(t-1) = \sum_{k=1}^{K} \alpha_{kit}^m \quad \forall t = 1, ..., T, \forall m, \forall i \in \bar{V}^m \cup \bar{\bar{V}}^m
\]  
(15)

\[
\beta_{it(t+1)}^m \geq \beta_{it}^m \quad \forall t = 1, ..., T - 1, \forall m, \forall i \in \bar{V}^m \cup \bar{\bar{V}}^m
\]  
(16)

\[
\sum_{t=1}^{p_{it}^m-1} \beta_{it}^m = 0 \quad \forall t = 1, ..., T, \forall m, \forall i \in \bar{V}^m \cup \bar{\bar{V}}^m
\]  
(17)

\[
\sum_{k=1}^{K} \sum_{t=1}^{p_{it}^m-1} \alpha_{kit}^m = 0 \quad \forall t = 1, ..., T, \forall m, \forall i \in \bar{V}^m \cup \bar{\bar{V}}^m
\]  
(18)

\[
\alpha_{kit}^m, \beta_{it}^m \in \{0,1\} \quad \forall t = 1, ..., T, \forall m, \forall i \in \bar{V}^m \cup \bar{\bar{V}}^m, \forall k = 1, ..., K
\]  
(19)

The civil infrastructure interdependency constraints determine whether a node is disrupted or not based on if the necessary services are being provided to it. For example, if there is a transshipment node that requires power to operate and it is not getting power, then no flow can pass through that transshipment node. In these constraints, \( y_{m,i}^{n,j,t} \) is a binary variable that is equal to 1 if the slack at the parent node \(i\), which is in infrastructure \(m\), is zero and thus the child node \(j\), which is in infrastructure \(n\), is functioning at time \(t\).

\[
d_{it}^m - v_{it}^m \leq (1 - y_{m,i}^{n,j,t})(d_{it}^m) \quad \forall t = 1, ..., T, \forall (i,j) \in F(m,n), j \in V^m, i \in V^{m,-}
\]  
(20)

\[
\sum_{(h,j) \in E^m} x_{hj}^m \leq s_j y_{m,i}^{n,j,t} \quad \forall t = 1, ..., T, \forall (i,j) \in F(m,n), j \in V^{m,+}, i \in V^{m,-}
\]  
(21)
\[ \sum_{(h,j) \in E} x_{hjt}^n \leq d_{j}^m y_{mj}^{n,ft} \forall t = 1, ..., T, \forall (i,j) \in F(m,n), j \in V^{n-}, i \in V^{m-} \tag{22} \]

\[ \sum_{(h,j) \in E} x_{hjt}^n \leq u_{j}^m y_{mj}^{n,ft} \forall t = 1, ..., T, \forall (i,j) \in F(m,n), j \in V^{n+}, i \in V^{m-} \tag{23} \]

\[ y_{mj}^{n,ft} \in \{0,1\} \forall t = 1, ..., T, \forall (i,j) \in F(m,n) \tag{24} \]

### 2.1.2 Objective Function

This section describes the objective function for the CRISIS model. While notation is described in this section, a summary of the entire notation set for the CRISIS model can be referenced in Appendix III. Equation (25) gives the overall objective function for this model.

\[
\text{maximize} \sum_{s \in S^N} W^s \cdot \Pi^s + \sum_{s \in S^D} W^s \cdot \Pi^s + \sum_{s \in S^G} W^s \cdot \Pi^s \tag{25}
\]

In this equation, \( \Pi^s \) is the overall performance of social infrastructure system \( s \). This function has three terms for the three different social infrastructure classifications. \( S^N \) is the set of systems with the emergency response network classification, \( S^D \) is the systems with the distribution network classification, and \( S^G \) is the systems with the affiliated set classification. For all three classifications, each system \( s \) is weighted independently by \( W^s \). Weighting each system serves two purposes. First, it provides the decision maker the opportunity to give priority of one system’s performance over the others. For example, if a decision maker wants to determine the optimal restoration plan to maximize the performance of the EMS system only, then the EMS system would be given a weight of 1, while all other systems are given a weight of 0. The values for \( W^s \) are also used for normalization so that each infrastructure system contributes equally to the objective. A procedure for calculating these weights is given in the application section. The objective terms for each of the three social infrastructure classifications in the CRISIS model are presented after each corresponding constraint set in the following sections.

### 2.1.3 The Emergency Response Network Classification

This section contains the details of the emergency response network classification of social infrastructure systems, which include police, fire, and EMS services. Let \( s \in S^N \) represent a social infrastructure system in the set of all emergency response network systems \( S^N \). Let \( V^{s+} \) be the set of supply nodes and \( V^{s-} \) the set of demand nodes for system \( s \). For each demand node \( i \in V^{s-} \), \( d_{iti}^s \) will be the normal demand level for service \( s \). \( D_{iti}^s \) is the adjusted demand level for service \( s \) in time period \( t \). Demand can be adjusted as the result of damage or disruption to node \( i \). This process is detailed later in
this section. For each supply node \( i \in V^s \), \( s_i^s \) will be the normal supply level for service \( s \) and \( S_{it}^s \) will be the adjusted supply level for service \( s \) in time period \( t \). Supply is affected in a similar way as demand, by damage or disruption.

One of the contributions of this model is the capability to delineate disutilities, i.e. costs, in the emergency response systems. These disutilities include penalizing the amount of time that is required for the emergency services to travel the transportation network from supply to demand and the penalty for unmet demand.

“The first objective of emergency rescue is to minimize the arrival time of rescue vehicles with sufficient resources. The economic cost is also concerned in emergency rescue; nevertheless, the economic cost is relatively less important than time efficiency.” (Hui et al. 2010)

Since all of the emergency services flow through the transportation network, they all share the same set of arcs and transshipment nodes. This model assumes that there are two different types of damage that can occur to any node or arc. Nodes and arcs with a high level of damage are represented by \( \bar{V}^s \) and \( \bar{E}^m \) respectively, while arcs and node with a low level of damage are represented by \( \bar{V}^s \) and \( \bar{E}^m \) respectively. The entire set of nodes and arcs are given by \( V^s \) and \( E^m \) respectively. Transshipment nodes, or intersections in this case, are given by \( V^{m=} \). In this formulation, transportation components with high damage are impassable, while components with low damage are passable, but have a higher cost associated with them when compared to undamaged components.

Let \( c_{ijz} \) be the time cost of traversing arc \( (i, j, z) \). The index \( z \) represents the amount of congestion on an arc; for this research \( z \) can take either the value of 1 or 2. Arcs represented with \( z = 1 \) have normal congestion and therefore have a lower value for \( c_{ijz} \). Arcs represented with \( z = 2 \) have higher congestion and thus have a higher value for \( c_{ijz} \). Arcs represented with \( z = 1 \) cannot be used if the actual arc or subsequent intersection is experiencing low damage or disruption. Arcs represented with \( z = 1 \) and \( z = 2 \) cannot be used if the real arc or subsequent intersection is experiencing high damage.

Figure 2 below is a visualization of this cost structure for an arbitrary transportation network. As depicted in this figure, arcs \( (s,c),(s,a) \) and \( (f,t) \) have normal congestion, arc \( (c,d) \) has high congestion as the result of partial damage to the arc, \( (d,f) \) and \( (b,f) \) have high congestion as the result of partial damage to intersection \( f \) and arc \( (a,b) \) is impassable as the result of high damage to the arc.
Fig. 2 Emergency Response Cost Structure

The first set of constraints is the network flow constraints. These constraints govern how the emergency services flow through the transportation network. Since the capacities on the transportation arcs are independent of the type of congestion, the flows for both \( z = 1 \) and \( 2 \) are combined in the capacity constraints. These constraints also state that the amount of demand flowing into a demand point, \( v_{it}^s \), cannot exceed the level for adjusted demand \( D_{it}^s \).

\[
\sum_{(i,j) \in E^m} \sum_{x \in \{1,2\}} x_{ijat}^s - \sum_{(j,i) \in E^m} \sum_{x \in \{1,2\}} x_{jiat}^s \leq S_{it}^s \quad \forall i \in V^{s,+}, m = \text{trans}, t = 1, \ldots, T, \forall s \in S^N
\]

(26)

\[
\sum_{(i,j) \in E^m} \sum_{x \in \{1,2\}} x_{ijat}^s = 0 \quad \forall i \in V^{m,x}, m = \text{trans}, t = 1, \ldots, T, \forall s \in S^N
\]

(27)

\[
\sum_{(i,j) \in E^m} \sum_{x \in \{1,2\}} x_{jiat}^s = -v_{it}^s \quad \forall i \in V^{s,-}, m = \text{trans}, t = 1, \ldots, T, \forall s \in S^N
\]

(28)

\[
0 \leq v_{it}^s \leq D_{it}^s \quad \forall i \in V^{s,-}, t = 1, \ldots, T, \forall s \in S^N
\]

(29)

\[
0 \leq \sum_{s \in S^N} \sum_{(j,i) \in E^m} x_{jiat}^s \leq u_{im}^m \quad \forall i \in V^s \cup V^{m,x}, m = \text{trans}, t = 1, \ldots, T
\]

(30)

\[
0 \leq \sum_{s \in S^N} \sum_{x \in \{1,2\}} x_{ijat}^s \leq u_{ij}^m \quad \forall (i,j) \in E^m, m = \text{trans}, t = 1, \ldots, T
\]

(31)
The next set of constraints manages the effects of damage to the transportation network for both nodes and arcs. The binary decision variables $\beta^{m}_{ijt}$ and $\beta^m_{jst}$ represent whether transportation arc $(i, j)$ and intersection $j$ respectively has been repaired prior to time period $t$. According to these constraints, arcs which exist in $\hat{E}^s$ cannot allow flow to pass through either arcs represented by $z = 1$ or $2$ while $\beta^{m}_{ijt} = 0$, while arcs that exist in $E^s$ cannot allow flow to pass through only arcs represented by $z = 1$ while $\beta^m_{jst} = 0$. Nodes that exist in $\hat{\overline{V}}^s$, cannot allow flow into or out of for arcs represented with $z = 1$ or $2$ while $\beta^m_{jst} = 0$, while nodes that exist in $\overline{V}^s$ cannot allow flow into or out of for arcs represented with $z = 1$ while $\beta^m_{jst} = 0$.

$$0 \leq x^s_{ijt} \leq u^m_{ijt} \beta^m_{ijt} \quad \forall s \in S^N, \forall (i, j) \in \hat{E}^m, z = 1, m = \text{trans}, \forall t = 1, ..., T$$  (32)
$$0 \leq x^s_{ijt} \leq u^m_{ijt} \beta^m_{ijt} \quad \forall s \in S^N, \forall (i, j) \in \overline{E}^m, z = \{1, 2\}, m = \text{trans}, \forall t = 1, ..., T$$  (33)
$$0 \leq x^s_{jst} \leq u^m_{jst} \beta^m_{jst} \quad \forall s \in S^N, \forall j \in \overline{E}^m, z = 1, m = \text{trans}, \forall t = 1, ..., T$$  (34)
$$0 \leq x^s_{jst} \leq u^m_{jst} \beta^m_{jst} \quad \forall s \in S^N, \forall j \in \overline{V}^m, z = \{1, 2\}, m = \text{trans}, \forall t = 1, ..., T$$  (35)

The next set of constraints determines what effect damage has on the supply and demand levels for emergency services. Factors such as damage to a critical facility can affect the need for services or the amount of resources available to use. For example, if a fire station is damaged or does not have water services, then the services that can be provided from the station would be limited. $\beta^s_{jst}$ is the variable that defines whether node $j$ in social infrastructure system $s$ in damaged in time period $t$. $\sigma^s_i$ is the percentage increase in demand as the result of low damage to demand node $i$ and $2 \times \sigma^s_i$ as the result of high damage. Supply reacts in the same manner as demand with the values for $\tau^s_i$ and $2 \times \tau^s_i$ except that the supply level usually drops as a result of damage as opposed to increases. $\sigma^s_i$ and $\tau^s_i$ can hold values anywhere in the range from $\{0.5, ..., \infty\}$. If this value is negative, it means that the value for demand or supply decreases as a result of damage. If the value is positive, it means that the value for demand or supply increases as a result of damage. The range begins at -0.5 because it does not allow the value for adjusted demand or supply to be negative.

$$D^s_{jt} \geq d_j^s + (2 \times \sigma^s_i d_j^s)(1 - \beta^s_{jst}) \quad \forall s \in S^N, \forall j \in \overline{V}^s, \forall t = 1, ..., T$$  (36)
$$D^s_{jt} \geq d_j^s + (\sigma^s_i d_j^s)(1 - \beta^s_{jst}) \quad \forall s \in S^N, \forall j \in \overline{V}^s, \forall t = 1, ..., T$$  (37)
$$S^s_{jt} \leq s_j^s + (2 \times \tau^s_i s_j^s)(1 - \beta^s_{jst}) \quad \forall s \in S^N, \forall j \in \overline{V}^s, \forall t = 1, ..., T$$  (38)
$$S^s_{jt} \leq s_j^s + (\tau^s_i s_j^s)(1 - \beta^s_{jst}) \quad \forall s \in S^N, \forall j \in \overline{V}^s, \forall t = 1, ..., T$$  (39)
$$D^s_{jt} \geq d_j^s \quad \forall s \in S^N, \forall j \in V^s, \forall t = 1, ..., T$$  (40)
\[ S^s_{jt} \leq s^s_j \quad \forall s \in S^N, \forall j \in V^s, \forall t = 1, ..., T \quad (41) \]

Demand and supply nodes can also be affected by disruption of civil infrastructure services. The following constraints determine the effects that disruption of civil infrastructure services has on components of the emergency response network social infrastructure systems. The assumption is that disruptions in any or all of the civil infrastructure systems can have an effect on the emergency response systems. Whether the particular node is a demand, supply, or transshipment node will determine the effect that a disruption has. \( y^{s,j,t}_{m,i} \) is defined as a binary variable which represents the slack that exists from civil infrastructure \( m \) to node \( j \) in social infrastructure \( s \). This node requires service from infrastructure \( m \) where it is identified as demand node \( i \). From the first constraint below, \( y^{s,j,t}_{m,i} \) is 1 if all demand for service \( m \) is being met at node \( i \) and 0 otherwise. The additional constraints show how supply, demand, and capacity are affected at node \( j \) in social infrastructure \( s \) as a result of this disruption.

\[
d^m_{it} - v^m_{it} \leq (1 - y^{s,j,t}_{m,i})(d^m_{it}) \quad \forall t = 1, ..., T, \forall (i,j) \in F(m, s), j \in V^s, i \in V^{m,-} \quad (42)
\]

\[
D^s_{jt} \geq d^s_j + \sum_{(i,j) \in F(m, s)} (\sigma^s_{d}d^s)(1 - y^{s,j,t}_{m,i}) \quad \forall t = 1, ..., T, j \in V^s, i \in V^{m,-} \quad (43)
\]

\[
S^s_{jt} \leq s^s_j + \sum_{(i,j) \in F(m, s)} (\tau^s_{s}d^s)(1 - y^{s,j,t}_{m,i}) \quad \forall t = 1, ..., T, j \in V^{s,+}, i \in V^{m,-} \quad (44)
\]

\[
\sum_{(h,j) \in E \cup \bar{E}^s} x^s_{hjwt} \leq u^s_{hj} y^{s,j,t}_{m,i} \quad \forall t = 1, ..., T, \forall (i,j) \in F(m, s), j \in V^{s,+}, i \in V^{m,-}, z = 1 \quad (45)
\]

The last set of constraints is the decision variable constraints which define the bounds on the decision variables.

\[
x^s_{jxt} \text{ integer} \quad \forall (i,j) \in E^m, s \in S^N, \forall t = 1, ..., T \quad (46)
\]

\[
v^m_{it} \text{ integer} \quad \forall i \in V^s, s \in S^N, \forall t = 1, ..., T \quad (47)
\]

\[
\beta^m_{it} \text{ binary} \quad \forall j \in V^m \cup \bar{V}^m, \forall m \in M, \forall t = 1, ..., T \quad (48)
\]

\[
\beta^m_{jx} \text{ binary} \quad \forall (i,j) \in E^m \cup \bar{E}^m, \forall m \in M, \forall t = 1, ..., T \quad (49)
\]

\[
\beta^s_{jt} \text{ binary} \quad \forall j \in V^s \cup \bar{V}^s, s \in S, \forall t = 1, ..., T \quad (50)
\]

\[
D^s_{it} \text{ integer} \quad \forall i \in V^s, s \in S, \forall t = 1, ..., T \quad (51)
\]

\[
S^s_{it} \text{ integer} \quad \forall i \in V^{s,+}, s \in S^N, \forall t = 1, ..., T \quad (52)
\]

\[
y^{s,j,t}_{m,i} \text{ binary} \quad \forall (i,j) \in F(m, s), \forall t = 1, ..., T \quad (53)
\]

Equation (54) is the objective for the systems in the emergency response network classification.
\[ \Pi^s = \sum_{t=1,T} \left( HW \cdot \sum_{i \in V^s} w_i^s \cdot (v_{it}^s - D_{it}^s) - \sum_{(l,j) \in E^s \cup E^s_z \cap \{1,2\}} c_{l,jz} x_{l,jz}^s \right) \quad \forall s \in S^N \]  

(54)

The first term in this equation is the penalty for not meeting the demand for emergency services and the second term is the sum of the amount of time cost that the emergency services uses in responding to the demand. In the first term, \( D_{it}^s \) is the adjusted demand level for service \( s \) at node \( i \) in time period \( t \) and \( v_{it}^s \) is the amount of demand met at node \( i \) in time period \( t \). Each demand node is weighted independently by \( w_i^s \). For the second term, \( x_{l,jz}^s \) is the flow of service \( s \) on arc \((i,j,z)\) in time period \( t \) while \( c_{l,jz} \) is the time cost of traversing arc \((i,j,z)\). In order to balance the two terms, and since it is important that the demand is met even at a high cost, a moderately heavy weight \( HW \) is put on the term for unmet demand. This weight should be just large enough to ensure that meeting demand is always the priority over the cost of meeting the demand.

### 2.1.4 The Distribution Network Classification

The distribution network classification is the second set of social infrastructure systems being modeled. Examples of these systems include fuel distribution and personal banking. Both of these systems are critical to individuals before and after a disaster. As seen in the quote below, the availability and access of cash and credit systems are critical for the wellbeing of individuals following a disaster.

"Customers and employees remaining in, or evacuating from, affected areas may need unexpectedly large amounts of cash to pay for critical goods and services." (Federal Financial Institutions Examination Council 2008)

The availability of the supply and distribution mechanisms of fuel is also critically important following an extreme event. Fuel is used to power automobiles, generators, and to perform tasks such as cooking food and heating homes.

"In the days immediately following (Hurricane Sandy), millions of customers remained without power; gas stations saw lines that were, in some cases, miles long. The disruptions to the system began well "upstream" from the gas station." (National Association of Convenience Stores 2013)

As seen during recent events such as Hurricane Sandy, these systems are as vulnerable to disruption as other social infrastructure systems, specifically from power and telecommunications outages (Federal Financial Institutions Examination Council 2008). The key difference between the distribution network and emergency response network classifications is that in the distribution network, services flow from supply points to distribution centers where they are obtained by the customers. For example, in the...
fuel distribution network, fuel is stored at fuel depots before it is distributed to fueling stations where individuals must travel in order to receive services. It is not only the accessibility of civil infrastructure services to supply points such as the transportation system to convey customers and the power and communications systems, but it is also the inflow of cash and fuel into the supply points that is crucial.

For this classification, the availability and movement of supply from supply depots to distribution points, as well as the demand and movement of customers to those distribution points are being modeled. Additional notation is needed to model this classification. Let \( s \in S^D \) represent a social infrastructure system in the set of all distribution network systems \( S^D \). Let \( V^s \) be the set of distribution points in system \( s \). Let \( DC^s_i \) be the delivery cost associated with the delivery of resources to distribution point \( i \). In this system, since there is movement of both supplies from supply points and customers from demand points, two types of resources must be defined. Let \( s1 \) be the supplies that move from a supply point to a distribution center and let \( s2 \) be the demand of the customers that moves from a demand point to a distribution center.

The first set of constraints is the supply, demand, and transshipment constraints. These constraints determine how much supply can leave the depots and how much demand can leave the demand points.

\[
\sum_{(i,j) \in E^m} x_{ijt}^{s1} \leq S_{it}^{s1} \quad \forall t = 1, \ldots, T, \forall i \in V^{s+}, m = \text{trans}, \forall s \in S^D \tag{55}
\]

\[
\sum_{(i,j) \in E^m} x_{ijt}^{s2} = 0 \quad \forall t = 1, \ldots, T, \forall i \in V^{s+}, m = \text{trans}, \forall s \in S^D \tag{56}
\]

\[
\sum_{(i,j) \in E^m} x_{ijt}^{s2} \leq D_{it}^{s2} \quad \forall t = 1, \ldots, T, \forall i \in V^{s-}, m = \text{trans}, \forall s \in S^D \tag{57}
\]

\[
\sum_{(i,j) \in E^m} x_{ijt}^{s1} = 0 \quad \forall t = 1, \ldots, T, \forall i \in V^{s-}, m = \text{trans}, \forall s \in S^D \tag{58}
\]

\[
\sum_{(i,j) \in E^m} x_{ijt}^{s1} - \sum_{(j,i) \in E^m} x_{jit}^{s1} = 0 \quad \forall t = 1, \ldots, T, \forall j \in V^{s-}, m = \text{trans}, \forall s \in S^D \tag{59}
\]

\[
\sum_{(i,j) \in E^m} x_{ijt}^{s2} - \sum_{(j,i) \in E^m} x_{jit}^{s2} = 0 \quad \forall t = 1, \ldots, T, \forall j \in V^{s+}, m = \text{trans}, \forall s \in S^D \tag{60}
\]

The next set of constraints is the distribution constraints. These constraints govern the operations of the distribution centers. Let \( \omega_{jt}^{s1} \) be the remainder of resource \( s \) left over at node \( j \) into time period \( t \). These constraints state the amount of demand met at a distribution center cannot exceed the amount of resources delivered in that time period plus the remainder from the previous time period. Also, the amount of resources at a distribution center at a given time cannot exceed its capacity and only a certain number of deliveries can be made within the time horizon. In this case, deliveries can only be made once
every $\theta^s_i$ time periods to a particular distribution center; in this case $T$ is the total number of time periods. $I^s_{it}$ is an indicator variable representing whether a delivery of service $s$ is made to distribution point $i$ in time period $t$. Also, in these constraints, $M$ is an appropriately large coefficient.

$$
\omega^{s1}_{j(t-1)} + \sum_{(i,j) \in E^m} x^{s1}_{ijt} - \sum_{(i,j) \in E^m} x^{s2}_{ijt} = \omega^{s1}_{jt} \quad \forall t = 2, \ldots, T, \forall j \in V^s, \forall s \in S^D
$$

(61)

$$
\sum_{(i,j) \in E^m} x^{s1}_{ijt} - \sum_{(i,j) \in E^m} x^{s2}_{ijt} = \omega^{s1}_{jt} \quad t = 1, \forall j \in V^s, \forall s \in S^D
$$

(62)

$$
\omega^{s1}_{j(t-1)} + \sum_{(i,j) \in E^m} x^{s1}_{ijt} \leq u^{s1}_{jt} \quad \forall t = 2, \ldots, T, \forall j \in V^s, \forall s \in S^D
$$

(63)

$$
\sum_{(i,j) \in E^m} x^{s1}_{ijt} \leq u^{s1}_{jt} \quad t = 1, \forall j \in V^s, \forall s \in S^D
$$

(64)

$$
\omega^{s1}_{jt} \geq 0 \quad \forall t = 1, \ldots, T, \forall j \in V^s, \forall s \in S^D
$$

(65)

$$
M \cdot (1 - I^s_{jt}) \geq \sum_{(i,j) \in E^m} x^{s1}_{ijt} \quad \forall t = 1, \ldots, T, \forall j \in V^s, \forall s \in S^D
$$

(66)

$$
\sum_{t=1}^{n} I^s_{jt} \geq \frac{T}{\theta^s_j} \quad \forall j \in V^s, \forall s \in S^D
$$

(67)

The next set of constraints is the interdependency constraints. Constraints (68) and (69) state that if the amount of demand that is met at distribution center $j$ in system $s$ exceeds a threshold $TH^s_j$, then this activates a demand for service $r$ at node $j$. For example, if a line at a gas station becomes too long, then there will be a need for police to control the situation at that location. Constraints (70) and (71) state that if a node $j$ is experiencing outage of service $s$, then this activates a demand for service $r$ at that node. For example, if an intersection loses power, a demand is generated for police to manage traffic at that location. In these constraints, $\theta^s_{jr}$ is an indicator variable to determine whether or not service $r$ is needed at node $j$, where node $j$ exists in system $s$. The variable $\sigma^s_{jr}$ is used to define whether the particular relationship exists between the two systems at node $j$. In this case, $\sigma^s_{jr}$ is equal to 0 if no relationship exists and 1 otherwise.

$$
\sum_{(i,j) \in E^m} x^{s2}_{ijt} - TH^s_j \leq M \cdot \theta^s_{jr} \quad \forall t = 1, \ldots, T, \forall j \in V^s, \forall r \in S, \forall s \in S^D
$$

(68)

$$
D^r_{jt} \geq d^r_j \cdot \sigma^s_{jr} \cdot \theta^s_{jr} \quad \forall t = 1, \ldots, T, \forall j \in V^s, \forall r \in S, \forall s \in S^D
$$

(69)

$$
\sigma^s_{m,t} \leq M \cdot \theta^s_{jr} \quad \forall t = 1, \ldots, T, \forall j \in V^s, \forall r \in S, \forall s \in S^D
$$

(70)

$$
D^r_{jt} \geq d^r_j \cdot \sigma^s_{jr} \cdot \theta^s_{jr} \quad \forall t = 1, \ldots, T, \forall j \in V^s, \forall r \in S, \forall s \in S^D
$$

(71)
The final set of constraints is the damage constraints. The damage constraints state that if a node or arc is damaged in the transportation network, then it cannot be used in the distribution networks for either the movement of supply or demand.

\[ 0 \leq x_{ijt}^{s1}, x_{ijt}^{s2} \leq u_{ijt}^{m} \beta_{ijt} \quad \forall s \in S^{D}, \forall (i, j) \in \bar{E}^{m}, m = \text{trans}, \forall t = 1, \ldots, T \quad (72) \]

\[ 0 \leq x_{ijt}^{s1}, x_{ijt}^{s2} \leq u_{j}^{m} \beta_{jt} \quad \forall s \in S^{D}, \forall j \in \bar{V}^{m}, m = \text{trans}, \forall t = 1, \ldots, T \quad (73) \]

\[ x_{ijt}^{s1}, x_{ijt}^{s2} \text{ integer} \quad \forall (i, j) \in E^{m}, \forall s \in S^{D}, \forall t = 1, \ldots, T \quad (74) \]

\[ \omega_{jt}^{s1} \text{ integer} \quad \forall j \in V^{s}, \forall s \in S^{D}, \forall t = 1, \ldots, T \quad (75) \]

\[ I_{jt}^{s} \text{ binary} \quad \forall j \in V^{s}, \forall s \in S^{D}, \forall t = 1, \ldots, T \quad (76) \]

\[ \theta_{jt}^{s} \text{ binary} \quad \forall j \in V^{s}, \forall s \in S^{D}, \forall t = 1, \ldots, T \quad (77) \]

Equation (78) is the function for the performance \( \Pi^{S} \) of the systems in the distribution network classification.

\[ \Pi^{S} = \sum_{t=1}^{T} \left( \sum_{i \in V^{s}} \sum_{(i, j) \in E^{m}} w_{i}^{s} \cdot x_{ijt}^{s2} - \sum_{i \in V^{s}} I_{it}^{s} \cdot DC_{i}^{s} \right) \quad \forall s \in S^{D} \quad (78) \]

The first term in this equation represents the total amount of met demand. In this classification, the met demand is equal to the amount of flow of customer demand out of the demand nodes. This constraint set assumes that a customer can only leave a demand point if resources are available at a distribution center. In the same way as before, each demand node is weighted independently by \( w_{i}^{s} \). The second term in this equation is the cost of delivering resources to the distribution points. Let \( I_{it}^{s} \) be an indicator representing whether a delivery of resource \( s \) is made to distribution point \( i \) in time period \( t \). \( DC_{i}^{s} \) is the cost associated with a delivery of resource \( s \) to distribution point \( i \).

### 2.1.5 The Affiliated Set Classification

In this section are the details of the affiliated set classification for social infrastructure systems. These systems do not provide services through the movement of people or goods through a network. It is assumed that these systems either provide services at one location, provide services remotely (e.g., cyber), or do not provide any services during an extreme event but are still critical for the wellbeing of a community. For example, it is assumed that in the healthcare systems, that when patients arrive at a location, there is no need to then travel to another location. The same is true with shelters; it is assumed there is no need to move people between shelters once they arrive at one.
For this classification, despite the assumption that services are not moving between nodes, the nodes in a system may not be independent from one another. When an event, change, or decision is made at one node, it can have an effect on the other nodes in the system. Table 3 below describes the three types of relationships that can exist in this model between nodes in the affiliated set system classification.

<table>
<thead>
<tr>
<th>Type of Relationship</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand</td>
<td>If Node A is damaged or disrupted, then Node B experiences a change, + or -, in demand level for a specified service</td>
<td>When a nearby shelter (A) loses power, shelter B experiences an increase in number of occupants which leads to increased demand for services</td>
</tr>
<tr>
<td></td>
<td>If Node A is damaged or disrupted, then Node B experiences a change, + or -, in supply level for a specified service</td>
<td>When hospital A shuts down, nearby hospital B receives additional staffing and resources</td>
</tr>
<tr>
<td>Supply</td>
<td>If Node A is damaged or disrupted, then Node B experiences loss of functionality</td>
<td>When the airport is damaged or not operational, local travel services (shuttles, rental cars) cannot function</td>
</tr>
<tr>
<td>Function</td>
<td>If Node A is damaged or disrupted, there is no effect on Node B</td>
<td>When national monument A is damaged, national monument B can still operate properly</td>
</tr>
</tbody>
</table>

It is assumed that only one of these relationships can exist between two given nodes. The first set of constraints defines how the performance of each individual node is determined based on their current damage level. Previously, the variable $\pi^s_{it}$ was defined as the performance of node $i$ which exists in system $s$ during time period $t$. This variable can take any value between and including 0 and 1. In these constraints, if a component is experiencing low damage, their performance is assumed to decrease by a factor of $\psi^{s,low}$ and if they are experiencing high damage, their performance is assumed to decrease by a factor of $\psi^{s,high}$.

$$\pi^s_{it} \leq (1 - \psi^{s,low}) \cdot \beta^s_{it} \quad \forall i \in \bar{V}^s, \forall s \in S^G, \forall t = 1, \ldots, T \quad (79)$$

$$\pi^s_{it} \leq (1 - \psi^{s,high}) \cdot \beta^s_{it} \quad \forall i \in \bar{V}^s, \forall s \in S^G, \forall t = 1, \ldots, T \quad (80)$$

The next set of constraints defines how the four relationships in the table above are modeled. $\lambda^s_{ij}$ is the variable that determines whether a relationship exists between two nodes $i$ and $j$ and what service $s$ that relationship has an effect on. $\lambda^s_{ij}$ takes the value of the percentage change in supply, demand, or function depending on the type of relationship. For example, if nodes 1 and 2 in the healthcare infrastructure have a relationship where node 2 gains 25% EMS demand if node 1 is damaged, then
\( \lambda_{(1)(2)}^{(EMS)} \) takes the value of 0.25. The type of relationship is determined based on what set node \( j \) exists in in system \( s \). In this example, since node 2 is a demand node in the EMS system, the demand attribute is affected. Similarly, if nodes 3 and 4 have a relationship where node 4 loses 50% of its function if node 3 is damaged, then \( \lambda_{(3)(4)}^{(EMS)} \) takes the value of -0.50. In this case node 4 is neither a demand nor supply node in the EMS system. If no relationship exists then \( \lambda_{ij}^{s} \) has a value of 0. Having an increase in demand or a decrease in supply at a node does not necessarily mean that the performance of that node decreases; it only makes it more difficult to meet all necessary demand. However, if a node loses function, it does result in a decrease of performance for that node. It is important that the values for \( \lambda_{ij}^{s} \) are determined based on the fact that adjusted supply and performance cannot be less than 0.

\[
D_{jt} \geq d_{jt} + \sum_{i \in V^{s}} (d_{jt}^{i} \ast \lambda_{ij}^{s} \ast \beta_{it}^{s}) \quad \forall j \in V^{s-}, \forall s \in S^{G}, \forall t = 1, ..., T \tag{81}
\]

\[
S_{jt}^{s} \leq s_{jt}^{s} + \sum_{i \in V^{s}} (s_{jt}^{i} \ast \lambda_{ij}^{s} \ast \beta_{it}^{s}) \quad \forall j \in V^{s+}, \forall s \in S^{G}, \forall t = 1, ..., T \tag{82}
\]

\[
\pi_{jt}^{s} \leq 1 - \sum_{i \in V^{s}} (\lambda_{ij}^{s} \ast \beta_{it}^{s}) \quad \forall j \in V^{s=}, \forall s \in S^{G}, \forall t = 1, ..., T \tag{83}
\]

\[
0 \leq \pi_{jt}^{s} \leq 1 \quad \forall j \in V^{s}, \forall s \in S^{G}, \forall t = 1, ..., T \tag{84}
\]

Equation (85) is the function for the performance \( \Pi^{s} \) of the systems in the affiliated set classification

\[
\Pi^{s} = \sum_{t=1}^{T} \sum_{i \in V^{s}} w_{it}^{s} \ast \pi_{jt}^{s} \quad \forall s \in S^{G} \tag{85}
\]

There is only one term in this function, which is the sum of the performance of all nodes in each system in each time period. The variable \( \pi_{jt}^{s} \) is defined as the performance of node \( i \) in system \( s \) during time period \( t \). Once again, each individual node is weighted independently by \( w_{it}^{s} \). In the sections to follow, there are descriptions of the constraints associated with each classification in the CRISIS model.

The affiliated set classification concludes the description of the CRISIS model in its entirety. To summarize, the purpose of the CRISIS model is to assist in making decisions regarding the restoration of critical civil infrastructure systems following an extreme event in order to improve the performance of the social infrastructure systems. The following section will provide an example of how this model can be used in practice to assist in this decision making process.
3. Application: Restoration of CLARC County after a Hurricane

This section will discuss the application of the CRISIS model for a hurricane event in an artificial community called CLARC county. CLARC county was created out of a need for a robust and sharable dataset for infrastructure and emergency management research. To our knowledge, this is the first attempt at developing an openly available artificial dataset for the purpose of studying interdependent civil infrastructure systems. This section will summarize key features of this dataset, however a complete description of the dataset can be found in the Ph.D. thesis of the corresponding author (Loggins 2015).

The structure of the dataset is that of a Geographic Information System (GIS) database. The CLARC county dataset is a representation of a hurricane prone coastal community of approximately 500,000 people and 1,065 square miles. This community is divided into 75 census tracts and includes the attributes associated with census tracts such as population, average income, terrain type, and the Social Vulnerability Index (SoVI®; Cutter et al. 2003) associated with the people in that area. Figure 3 below shows the design of the county and how the census tracts are divided. The colors of the census tracts represent what type of terrain is most prevalent in that area.

![Fig. 3 CLARC County Census Tracts](image)

The southern and eastern sides of the county border the ocean, while the northern and western sides share a border with other counties. Within each one of these census tracts, there also exists a set of facilities that are deemed critical by the emergency management community. Figure 4 below shows the southwest portion of the county, and the critical facilities that exist within that region.
In addition to critical facility and census data, the dataset also contains five civil infrastructure systems which include power, telecommunications, drinking water, waste water, and transportation. All of these systems were built based upon realistic design guidelines, the distribution of the population in the county, and the location of the critical facilities. In addition to each individual infrastructure system, the connections of interdependencies between the systems were also included in the dataset.

Figure 5 below shows the design of the CLARC county power system. This power system is comprised of nodes such as power generating plants and substations, and arcs such as transmission lines. In this figure, the arrows on the transmission lines show the typical direction of flow on those lines.
Figures 6 and 7 below show the results of the damage assessment model presented in Loggins and Wallace (2015) for a category 3 hurricane. In these maps, damage scenarios for the power and transportation infrastructure systems are shown. While these maps show only the expected damage for two of the systems being considered, in the subsequent analysis, the damage to all of the infrastructure systems and critical facilities produced by this model are considered.
Fig. 6 CLARC County Power Expected Damage Map Example

Fig. 7 CLARC County Transportation Expected Damage Map Example
In addition to physical damage, the CRISIS model also accounts for the disruption of services that is caused by that damage. This disruption is calculated using the service disruption model presented in the same paper as the damage assessment model (Loggins and Wallace 2015). Figure 8 shows the critical facilities and infrastructure components in CLARC county that are, and are not, experiencing power service outages as a result of the damage scenario above. If a component or facility does not require power services, then it does not appear on this map. As a result of the interdependencies that exist in the data, these outages are not solely as a result of damage directly to power components, but also resulting from damage to other infrastructure systems upon which the power system depends.

![Fig. 8 CLARC County Power Expected Disruption Map](image)

After the damage and disruption assessment is conducted, the next step is to determine the weights in the objective function for the CRISIS model. There are three types of weights that exist in this function. The first type is the weights on the individual demand nodes. These weights are predetermined based on the priorities of the decision maker. For example, if a hospital has a higher priority than a factory, then the hospital would be given larger weight. The second type of weight was the weight placed on unmet demand in the emergency response network classification. This weight was chosen to be the sum of all of the time costs in the transportation network. This ensures that there would never be a situation where an emergency responder would not respond to a call because the distance was too great. The final set of weights is the weights on each individual social infrastructure system. The procedure for
determining these weights is given here. First the CRISIS model is run for a case where there is no damage to any infrastructure components; this results in the best performance, $\Pi^s$, possible for each system. The next step is to determine the values for relative performance, $\bar{\Pi}^s$, of each system when compared to the system with the largest value. Equation (86) gives the formula for relative performance.

$$\bar{\Pi}^s = \frac{\max \left( \Pi^s \right) \forall s \in S}{\Pi^s}$$  \hspace{1cm} (86)

After the value of $\bar{\Pi}^s$ is determined for each system, the weight, $W^s$, for each system is calculated by the following equation.

$$W^s = \frac{\bar{\Pi}^s}{\sum_{s \in S} \bar{\Pi}^s}$$  \hspace{1cm} (87)

Table 4 below gives the normalized weights for all ten of the social infrastructure systems in CLARC county.

**Table 4 Normalized Weights for CLARC County Social Infrastructure Systems**

<table>
<thead>
<tr>
<th>Infrastructure System</th>
<th>Police</th>
<th>Fire</th>
<th>EMS</th>
<th>Fuel</th>
<th>Banking</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Best Performance</strong></td>
<td>25,370</td>
<td>12,685</td>
<td>6,342</td>
<td>590</td>
<td>2,114</td>
</tr>
<tr>
<td><strong>Normalized Weight</strong></td>
<td>0.12%</td>
<td>0.25%</td>
<td>0.49%</td>
<td>5.30%</td>
<td>1.48%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Infrastructure System</th>
<th>Healthcare</th>
<th>Residential</th>
<th>Industry</th>
<th>Education</th>
<th>Travel</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Best Performance</strong></td>
<td>604</td>
<td>116</td>
<td>97</td>
<td>258</td>
<td>189</td>
</tr>
<tr>
<td><strong>Normalized Weight</strong></td>
<td>5.17%</td>
<td>26.72%</td>
<td>31.90%</td>
<td>12.07%</td>
<td>16.50%</td>
</tr>
</tbody>
</table>

After the weights on each system are calculated, the CRISIS model can be run to determine the optimal restoration plan for the civil infrastructure systems. This is conducted to maximize the performance of the social infrastructure systems while accounting for the interdependencies that exist among these systems. For this example, the number of available work crews is set to 2 for each component type such as power arcs, power nodes, water arcs, and water nodes. Figures 9 and 10 below show the results of the CRISIS model. While only the restoration plans for power and transportation are being presented here, all five civil infrastructure systems are being restored in this example. In these figures, the blue numbers represent the time period the restoration of that particular node or arc is
completed. In this example, each time period represents a six-hour interval and the total time horizon includes 20 time periods. This results in a five-day planning horizon from when the storm makes landfall.

Fig. 9 CLARC County Power Restoration Example

Fig. 10 CLARC County Transportation Restoration Example
After the restoration plan is generated for the civil infrastructure systems, an analysis of what effect the restoration plan has on the social infrastructure systems they serve can be conducted. Figure 11 below shows the performance levels of multiple social infrastructure systems including the three emergency response network systems, two distribution network systems, and two of the affiliated set social infrastructure systems. The results are given over the first 15 time periods of the time horizon. It is not shown for the entire 20 time periods because most systems show insignificant change over that time.

![Graph showing social infrastructure performance results](image)

**Fig. 11 CRISIS Model, Social Infrastructure Performance Results**

As seen in this chart, after the event, the initial performance of these systems is significantly diminished; 72% for EMS, 80% for police, 66% for fire, 0% for banking, 10% for fuel, 56% for healthcare, and 52% for education. As restoration is being performed, the performance of these systems steadily increases over the time horizon. In this example, only the police and education infrastructure systems return to full performance prior to the 15th time period. At this time, the EMS system recovers to about 93% performance, while the banking system only returns to approximately 70% performance. The only irregularity in this chart is that the banking infrastructure experienced a performance drop in the second time period. Upon investigation, this was the result of a lack of resources at multiple distribution points during that time period because they were not accessible from the supply centers. More specifically, the road network around the primary banking center was heavily damaged, therefore the mechanisms for replenishing ATMs was not available.

In addition to the performance of each social infrastructure system as a whole, each individual node has a performance level. Figure 12 below shows the performance of each individual critical facility in CLARC county at the beginning of the time horizon and Figure 13 shows the performance after restoration has been completed. Performance is measured on a scale of 0% to 100%. As seen in these figures, there are obvious geographic trends for performance within the county. This is due to the fact that
damage and disruption are often collocated and services are not typically restored to one facility at a time, but restored to pockets of facilities and areas.

The results of the CRISIS model will now be compared with the results from the IINDS model presented by Cavdaroglu et al. (2013). As previously discussed, the IINDS model considers the restoration of interdependent civil infrastructure systems in a similar way as the CRISIS model; however it does not account for the performance of the social infrastructure systems. As a result, it can be shown
that the performance of these social infrastructure systems will benefit from using the CRISIS model over the previous IINDS model.

The following charts show how the civil and social infrastructure systems perform subject to the decision made in the two restoration plans given above. As seen in Figure 14, the performance of the emergency response network social infrastructure systems increases at a significantly greater rate in the CRISIS model over the IINDS model. While it is likely that the performance levels for each model will converge at some point, eventually at 100%, the performance of these systems during the restoration process is also very important because people rely on emergency services during the time immediately following the event.

**Fig. 14 CRISIS and IINDS Models Social Results Comparison**

In Figure 14, after 15 time periods both the IINDS and CRISIS model have the fire system operating at the same performance as prior to the event. The EMS system is operating at 94% in the CRISIS model and 90% in the IINDS model, while the police system is operating at 86% in the CRISIS model and 83% in the IINDS model after 15 time periods.

Figure 15 compares the performance of three civil infrastructure systems between the CRISIS and IINDS models. The performance of these systems is measured by the total amount of demand that is met in each time period in the power, water, and telecom civil infrastructure systems. The wastewater infrastructure was not included because it performed similarly to the water infrastructure.
From this figure it is shown that the IINDS model has slightly better performance when it comes to providing these services over time. However, since the goal of this research was to investigate the impact of restoration by assuming that the performance of the social infrastructure systems is the best measure of recovery in a community, the CRISIS model outperforms the IINDS model in that case. The purpose of the civil infrastructure systems is to support the social infrastructure systems and, as a result, the performance of the social infrastructure systems should be the primary consideration when making decisions regarding infrastructure restoration. Therefore, the assumption is that the increase in performance in the social infrastructure systems more than compensates for the loss of performance in the civil infrastructure systems.

Several computational tests have been conducted to show that the models presented in this paper run in a timely and efficient manner. All tests have been conducted on a personal computer with the following specifications and software packages:

- Intel I-7, 2.4 GHZ processor, 8 GB of RAM
- IBM ILOG CPLEX Optimization Studio 12.5 with models coded in OPL
- MATLAB R2014a with models coded in VBA

Table 5 provides the results for the CRISIS model. This model is a mixed integer, multiple time period formulation. This table shows the results for nine tests of this model using the CLARC county dataset. These results represent the combination of three hurricane categories and three values for the number of work crews available.
Table 5 CRISIS Model Computational Results

<table>
<thead>
<tr>
<th>Run Description</th>
<th># of Damaged Arcs and Nodes</th>
<th># of Restored Nodes</th>
<th># of Restored Arcs</th>
<th>Computational Time (H:M:S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Work Crew, Category 1</td>
<td>31</td>
<td>1</td>
<td>3</td>
<td>00:02:45</td>
</tr>
<tr>
<td>1 Work Crew, Category 3</td>
<td>388</td>
<td>18</td>
<td>24</td>
<td>01:51:00</td>
</tr>
<tr>
<td>1 Work Crew, Category 5</td>
<td>1464</td>
<td>33</td>
<td>18</td>
<td>00:22:25</td>
</tr>
<tr>
<td>2 Work Crews, Category 1</td>
<td>31</td>
<td>1</td>
<td>3</td>
<td>00:14:42</td>
</tr>
<tr>
<td>2 Work Crews, Category 3</td>
<td>388</td>
<td>21</td>
<td>32</td>
<td>02:35:19</td>
</tr>
<tr>
<td>2 Work Crews, Category 5</td>
<td>1464</td>
<td>47</td>
<td>33</td>
<td>01:50:11</td>
</tr>
<tr>
<td>3 Work Crews, Category 1</td>
<td>31</td>
<td>1</td>
<td>3</td>
<td>00:30:11</td>
</tr>
<tr>
<td>3 Work Crews, Category 3</td>
<td>388</td>
<td>24</td>
<td>78</td>
<td>02:50:12</td>
</tr>
<tr>
<td>3 Work Crews, Category 5</td>
<td>1464</td>
<td>59</td>
<td>46</td>
<td>06:54:08</td>
</tr>
</tbody>
</table>

There are a few observations that can be derived from these results. First, as the hurricane category increases, the number of damaged arcs/nodes increases, therefore the number of potential candidates for restoration increases. In these results, if there was enough slack on work crew utilization, then the number of restored arcs/nodes increased, otherwise that number remained constant. Second, the amount of computational time significantly increased as both the hurricane category increased and as the number of work crews increased. The only exception to this is the test with 1 work crew and a category 5 storm. The reason why this time was relatively fast was because there was such a large amount of damage that the model determined quickly that 1 work crew was not enough to make productive repairs within the time horizon. This result could provide the decision maker with the insight as to how many work crews would be needed to make productive repairs given high levels of damage.

4. Summary and Conclusions

This paper presents a robust model related to the recovery of civil and social infrastructure systems following an extreme event. The CRISIS model is a mixed-integer programming formulation designed to provide guidance in the restoration of civil infrastructure systems in the context of response and recovery from an extreme event. Recovery is measured by the improvement in performance of the social infrastructure systems over the time horizon. The CRISIS model includes a classification schema used to identify social infrastructure systems with similar characteristics. There are several unique features of this model including the addition of dynamic demand and supply levels, which are functions of the damage and disruption in these systems, as well as routing considerations. In order to test this model, an artificial community dataset was developed called CLARC county. This dataset contains multiple civil and social infrastructure systems as well as hundreds of critical facilities.
While this research has made advances in the modeling of social infrastructure systems, there are still areas of research left unexplored. A great deal of emphasis in this research was to build realistic models of social infrastructure systems and to show how they can be applied to assist in the decision making process. One potential area to expand this research is the investigation of cyber and information infrastructures and their interdependencies with other civil and social infrastructure systems. Many infrastructure systems rely on cyber or information infrastructure systems such as finance, transportation, and commerce. Many civil infrastructures such as power and communications rely on cyber systems in the form of information technology (IT) and supervisory control and data acquisition (SCADA) systems (Ericsson 2010); however, the reverse is true as well. Cyber and information infrastructures almost always rely on power and communication systems in order to function (Laprie et al. 2007).

Two of the critical social infrastructure systems that were not discussed in this paper were commercial supply chain and disaster relief supply chain systems. In an emergency, both of these systems are responsible for distributing critical goods such as food, water, medicine, and blankets. When a disaster strikes, it is common for there to be major disruptions in the supply and availability of these critical goods (Gong et al. 2014). This could be as a result of loss of suppliers, damage to the road network, or disruption of services such as power or fuel. The objective of commercial supply chains is to return their operations back to normal as quickly as possible in order to both serve their community and their business interests. The objective of the disaster relief supply chain is to get set up as quickly as possible and in a way that will be able to serve the individuals in a community. There are similarities and differences between these systems and the distribution systems discussed in this paper.

This research had two objectives. The first objective was to understand infrastructure interdependencies and the effect that they have on a community after an extreme event. The second objective was to construct realistic and innovative models of social infrastructure systems to assist in the post-event restoration of these systems in a way that maximizes the resilience of a community. As seen in the results of this work and the applications provided in this paper, the models presented provide valuable insights into the decision making process regarding the restoration of social and civil infrastructure systems. The solutions provided by the models are improvements over previous work conducted and could benefit decision makers after a disaster.

5. References


6. Appendix I: Notation

Data Types
Float – can take the value of any real number
Float+ – can take the value of any real, positive number
Set – is a set of nodes or arcs
Int – can take the value of any positive integer
Bin – can take the values of 0 or 1

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameters</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Π𝑠</td>
<td>the aggregate performance of social infrastructure system s</td>
<td>Float+</td>
</tr>
<tr>
<td>π𝑖𝑠</td>
<td>the performance of node i ∈ 𝑉𝑠 in social infrastructure system s</td>
<td>Float+</td>
</tr>
<tr>
<td>𝑊𝑠</td>
<td>the weight on social infrastructure system s</td>
<td>Float+</td>
</tr>
<tr>
<td>𝑐𝑖𝑗</td>
<td>the time cost of traversing arc (i, j, z)</td>
<td>Float+</td>
</tr>
<tr>
<td>∀ 𝑠 ∈ 𝑆⊗</td>
<td>the set of emergency response networked social infrastructure systems</td>
<td>Set</td>
</tr>
<tr>
<td>∀ 𝑠 ∈ 𝑆𝐷</td>
<td>the set of distribution network social infrastructure systems</td>
<td>Set</td>
</tr>
<tr>
<td>∀ 𝑠 ∈ 𝑆𝐺</td>
<td>the set of affiliated set social infrastructure systems</td>
<td>Set</td>
</tr>
<tr>
<td>𝑉𝑠</td>
<td>the undamaged nodes in social network s</td>
<td>Set</td>
</tr>
<tr>
<td>𝜽𝑠</td>
<td>the fully damaged nodes in social network s</td>
<td>Set</td>
</tr>
<tr>
<td>𝐼𝑠</td>
<td>the supply nodes in the social network</td>
<td>Set</td>
</tr>
<tr>
<td>𝐷𝑠</td>
<td>the demand nodes in the social network</td>
<td>Set</td>
</tr>
<tr>
<td>𝐸𝑠</td>
<td>the transshipment nodes in the social network</td>
<td>Set</td>
</tr>
<tr>
<td>𝑠𝑖</td>
<td>the supply level for each supply node i ∈ 𝑉𝑠+</td>
<td>Int</td>
</tr>
<tr>
<td>𝑑𝑖</td>
<td>the demand level for each demand node i ∈ 𝑉𝑠−</td>
<td>Int</td>
</tr>
<tr>
<td>𝜂𝑖</td>
<td>the allowed frequency for deliveries from node i</td>
<td>Float+</td>
</tr>
<tr>
<td>𝑇𝐻𝑖</td>
<td>the threshold for requiring service r in system s</td>
<td>Int</td>
</tr>
<tr>
<td>𝐷𝐶𝑖</td>
<td>the delivery cost for service s from node i</td>
<td>Float+</td>
</tr>
<tr>
<td>𝜙</td>
<td>the percentage of performance decreases as the results of damage to a facility in system s</td>
<td>Float+</td>
</tr>
<tr>
<td>(V^m)</td>
<td>the nodes in the civil network</td>
<td>set</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>(i \in V^{m+})</td>
<td>the supply nodes in the civil network</td>
<td>set</td>
</tr>
<tr>
<td>(i \in V^{m-})</td>
<td>the demand nodes in the civil network</td>
<td>set</td>
</tr>
<tr>
<td>(i \in V^{m=})</td>
<td>the transshipment nodes in the civil network</td>
<td>set</td>
</tr>
<tr>
<td>((i,j) \in E^m)</td>
<td>the undamaged arcs in the civil network</td>
<td>Set</td>
</tr>
<tr>
<td>((i,j) \in \bar{E}^m)</td>
<td>the fully damaged arcs in the civil network</td>
<td>Set</td>
</tr>
<tr>
<td>((i,j) \in \tilde{E}^m)</td>
<td>the partially damaged arcs in the civil network</td>
<td>Set</td>
</tr>
<tr>
<td>(u_{ij}^m)</td>
<td>the capacity associate with arc ((i,j)) in infrastructure (m)</td>
<td>Int</td>
</tr>
<tr>
<td>(s_i^m)</td>
<td>the supply level for each supply node (i \in V^{m+})</td>
<td>Int</td>
</tr>
<tr>
<td>(d_i^m)</td>
<td>the demand level for each demand node (i \in V^{m-})</td>
<td>Int</td>
</tr>
<tr>
<td>(w_i^m)</td>
<td>the weight on demand for node (i \in V^{m-})</td>
<td>Float+</td>
</tr>
<tr>
<td>(P_{ij})</td>
<td>the processing time for installing arc ((i,j) \in \tilde{E}^m)</td>
<td>Int</td>
</tr>
</tbody>
</table>

**Decision Variables**

- \(x_{ijst}\): the flow of service \(s\) on arc \((i,j)\) in time period \(t = 1, \ldots, T\) Float+
- \(v_{it}\): the amount of demand met at \(i \in V^{m-}\) in time period \(t = 1, \ldots, T\) Float+
- \(s_{it}\): the adjusted supply level for supply node \(i \in V^{s+}\) in time period \(t = 1, \ldots, T\) Int
- \(D_{jt}\): the adjusted demand level for demand node \(j \in V^{s-}\) in time period \(t = 1, \ldots, T\) Int
- \(I_{it}\): is an indicator variable representing whether a delivery of resource \(s\) was made from node \(i\) in time period \(t\) Bin
- \(\beta_{jt}\): is an indicator variable representing whether service \(s\) is required at node \(j\) in time period \(t\) Bin
- \(\omega_{js}^t\): The remainder of resource \(s\) at node \(j \in V^{s+}\) after time period \(t = 1, \ldots, T\) Int
- \(x_{ijmt}\): the flow of civil service \(m\) on arc \((i,j)\) in time period \(t = 1, \ldots, T\) Float+
- \(v_{jt}\): the amount of demand met at \(j \in V^{m-}\) Float+
- \(\rho_{ijt}\): =1 if arc \((i,j)\) in civil infrastructure system \(m\) is available in time period \(t\), 0 otherwise Bin
- \(\rho_{jt}\): =1 if node \(j\) in civil infrastructure system \(m\) is available in time period \(t\), 0 otherwise Bin
- \(\rho_{jt}^s\): =1 if node \(j\) in social infrastructure system \(s\) is available in time period \(t\), 0 otherwise Bin
- \(\alpha_{ijkt}\): =1 if work crew \(k\) completes arc \((i,j)\) in time period \(t\), 0 otherwise Bin
- \(y_{mi}^n\): =1 if node \(j\) in infrastructure \(n\) is receiving all needed services from infrastructure \(m\) during time period \(t\), therefore node \(i\) has 0 slack, 0 otherwise Bin
- \(y_{mt}\): =1 if node \(j\) in social infrastructure \(s\) is receiving all needed services from civil infrastructure \(m\) during time period \(t\), therefore node \(i\) has 0 slack, 0 otherwise Bin