FP 1

FEASIBILITY PUMP (Fischetti et al.)

$$\begin{aligned} \min \quad & c^T x \\ \text{s.t.} \quad & Ax \geq b \qquad\qquad (MIP) \\ & 0 \leq x \leq u \\ & x_i \text{ integer}, \quad i \in I \end{aligned}$$

Want to use the solution x^* to the LP relaxation to find a good feasible solution to (MIP).

So round x^*, get $\bar{x}$.

But $\bar{x}$ might not be feasible. How to proceed?

Try to find a new point x^+ feasible in LP relaxation that is close to $\bar{x}$:

$$\begin{aligned} x^+ = \arg\min_{x} \quad & \sum_{i \in I} |x_i - \bar{x}_i| \\ \text{s.t.} \quad & Ax \geq b \\ & 0 \leq x \leq u. \end{aligned}$$

Note: use 1-norm, to encourage sparsity, which here means try to get x integral. (sparsity in $(x - \bar{x})$).

Then round x^+

Repeat.

If cycle: try a random perturbation.

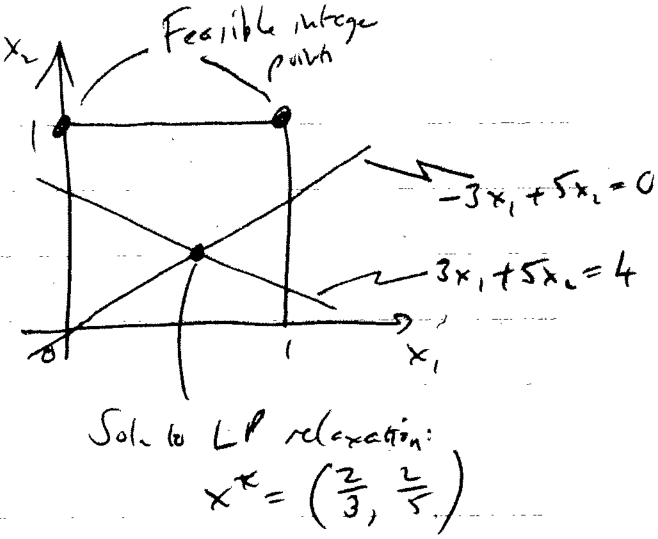
FP2

Eg: $\min \; x_1 + 2x_2$

s.t. $-3x_1 + 5x_2 \geq 0$

$3x_1 + 5x_2 \geq 4$

x_i binary



Get $x^* = \left(\frac{2}{3}, \frac{2}{5}\right)$

Round: $\bar{x} = (1, 0)$

Find closest point in relaxation:

$$x^* = \arg\min_x \; (1 - x_1) + (x_2 - 0)$$

$$\text{s.t. } -3x_1 + 5x_2 \geq 0$$

$$3x_1 + 5x_2 \geq 4$$

$$0 \leq x_1, x_2 \leq 1.$$

so $x^* = \left(1, \frac{3}{5}\right)$

Round again: $\bar{x} = (1, 1)$. Now feasible.

Note: $\bar{x}$ is not optimal.

Can add refinements to the method to improve quality of final $\bar{x}$.

Eg: update $x^* = \arg\min \; \alpha \|x - \bar{x}\|_1 + (1-\alpha)\, c^T x$ | Gradually increase

s.t. $Ax \geq b, \; 0 \leq x \leq u.$ | α if necessary.