Two exams

Two of the courses in which David is enrolled have their first exams next week. He is already confident that he knows 2 of the 5 textbook sections to be covered by the Linear Programming exam, but in dark moments of terror and self-reproach he is forced to admit that he has so far learned nothing at all about Art History. He estimates that he can master the remaining Linear Programming sections if he spends 3 hours studying the book and 2 hours working problems, but to catch up in Art History he needs to devote 10 hours to learning his class notes and visiting the on-line gallery. He hopes to get the highest grades he possibly can, but to avoid having an alert sent to his advisor he must score at least 60% on each exam. Unfortunately, his family commitments and other courses leave him only 12 hours to prepare for these exams. What should he do?

\[ s_1 = f(x_1) = 40 + 12x_1 \]
\[ s_2 = f(x_2) = 0 + 10x_2 \]
HW1 due: Friday September 13

Jia will have office hours on Thursdays

Time TBD
Decision variables:

\[ X_1 : \text{hours on LP} \]

\[ X_2 : \text{hours on Art History} \]

State variables:

Score on exams

\[ S_1 : \text{score in LP} \]

\[ S_2 : \text{score in Art History} \]
Max \( 40 + 12x_1 + 10x_2 \)

Subject to
\[
\begin{align*}
    x_1 + x_2 &\leq 12 \\
    40 + 12x_1 &\geq 60 \\
    10x_2 &\geq 60 \\
    40 + 12x_1 &\leq 100 \quad \text{(maximum score)} \\
    10x_2 &\leq 100 \\
    x_1, x_2 &\geq 0
\end{align*}
\]

\[ 40 + 12x_1 + 10x_2 = 170 \]
Brewery

When barley is allowed to partially germinate and is then dried, it becomes malt. When malt is crushed and mixed with water, boiled with hops, and fermented with yeast it becomes the delightful beverage we call beer. Sarah operates a local craft brewery that makes Porter, Stout, Lager, and India Pale Ale beer by using different amounts of pale malt, black malt, and hops. For example, to make 5 gallons of Porter requires 7 pounds of pale malt, 1 pound of black malt, and 2 ounces of hops, and the finished keg can be sold for $90. The technology table below summarizes the resource requirements and anticipated revenue for all four varieties, along with the stock on hand of each ingredient.

<table>
<thead>
<tr>
<th></th>
<th>Porter</th>
<th>Stout</th>
<th>Lager</th>
<th>IPA</th>
<th>stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>pale malt</td>
<td>7</td>
<td>10</td>
<td>8</td>
<td>12</td>
<td>160 lb</td>
</tr>
<tr>
<td>black malt</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>50 lb</td>
</tr>
<tr>
<td>hops</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td>60 oz</td>
</tr>
<tr>
<td>revenue</td>
<td>90</td>
<td>150</td>
<td>60</td>
<td>70</td>
<td></td>
</tr>
</tbody>
</table>

How much of each product should Sarah make to maximize her revenue?
Decision variables:

\[ x_1: \text{kegs of Porter} \]
\[ x_2: \text{kegs of Stout} \]
\[ x_3: \text{kegs of Lager} \]
\[ x_4: \text{kegs of IPA} \]

Maximize

\[ 90x_1 + 150x_2 + 60x_3 + 70x_4 \]

Subject to

\[ 7x_1 + 10x_2 + 8x_3 + 12x_4 \leq 160 \hspace{1cm} \text{(pale malt limit)} \]
\[ x_1 + 3x_2 + x_3 + x_4 \leq 50 \]
\[ 2x_1 + 4x_2 + x_3 + 3x_4 \leq 60 \]

\[ x_1, x_2, x_3, x_4 \geq 0 \]