Mathematical Modeling of Mechanoreceptors
Overview of the Ear

1. Eardrum
2. Malleus
3. Incus
4. Stapes
5. Semicircular canals
6. Auditory nerve
7. Vestibular nerve
8. Cochlea
9. Eustachian tube
The Cochlea
Inside the Cochlea
Mechno to Neural Step
Modeling: the geometry

- Oval window ($\Gamma_w$)
- Round window
- Helicotrema
- Basilar membrane
- Cochlear wall

- y
- x
- z
- B
- L
- $\Psi$
Fluid in Two Chambers

\[ (\partial_t - \delta^2 \nabla^2) \mathbf{v} = -\nabla p \]
\[ \nabla \cdot \mathbf{v} = 0 \]

Kinematic Boundary Condition

\[ v_z(x, y, 0, t) = \partial_t \eta(x, y, t) \]

Basilar Membrane

\[ (D \partial_y^4 + \mu \partial_t^2) \eta = -2p(x, y, 0, t) \]

\( \mathbf{v} = \) fluid velocity
\( p = \) fluid pressure
\( \eta = \) vertical displacement of BM
WKB Approximation

\[ \eta(x, y, t) \sim e^{i(k-\omega t)} A(x) \cos \left( \frac{\pi y}{2G} \right) \]

where

\[ A(x) \sim \beta \theta_x \exp \left( -\gamma \delta^2 \int_0^x (\theta_x)^2 \, dx \right) \]

\[ \theta_x \sim \frac{\alpha}{\Lambda(x) - \omega} \quad \Lambda(x) = \left( \frac{\pi}{2G} \right)^2 \sqrt{D \mu} \quad k = \frac{\theta(x)}{\epsilon} \]
Frequency = 400 Hz
Frequency = 1200 Hz
Experimental Results

Spatial Distribution

Fixed Spatial Location

Stimulus level (dB SPL)

Frequency (kHz)

2 kHz
3 kHz
0.8 kHz
5 kHz
1.5 Hz
7 kHz
20 kHz

'tail'
OHC loss
Normal
'tip'

2 kHz
What can’t be done (yet)

Experimental: can’t observe the motion throughout cochlea - only able to peek into the stapes end

Numerical: can’t compute the motion - current best effort uses over 100 cpu hours and 100 Gb memory and is still not able to solve problem

Conclusion: need more mathematical analysis
Open Questions

How to compute the transient problem?

What is role of the outer hair cells in the hearing process?
  Active hearing; nonlinear feedback

What about intensity coding?
  Combination tones

Is it possible to use inverse problem for diagnostic applications?
Courses

Math-4700  FOAM
Math-6620  Perturbation Methods
Math-6840  Numerical PDEs
Math-6640  Complex Variables and Transforms
Math-6600  MOAM