Math 2800 Intro to Discrete structures Spring 2009

FINAL EXAM PRACTICE PROBLEMS

__Note__ While these problems are typical of the type we've discussed, don't rely on the exam problems being __exactly__ like these. Make sure you're comfortable with the ideas in the problems, and don't be dependent on a specific form or wording. For a __complete__ list of potential topics, see web.

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① For each description, draw an example of a simple graph meeting it or explain why none can exist:

a) G is planar, not complete, and $\chi(G) = 4$

b) G's degree sequence is $\{5, 4, 4, 3, 2, 2\}$

c) G is planar and contains $K(3,3)$ as a subgraph.

d) G has 7 vertices, is non-planar, and does __not__ contain K_5 as a subgraph.

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② Solve the recurrence : $a_n = a_{n-1} + 2a_{n-2}$; $a_0 = 1$, $a_1 = 0$

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③ Use mathematical induction to prove that

$$\sum_{k=1}^{n} k2^k = 2 + (n-1)2^{n+1}, \quad \text{for } n = 1, 2, 3, \ldots$$

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④ show that the proposition $\neg p \to p$ is logically equivalent to p.

⑤ a) Use the euclidean algorithm to compute $\gcd(198, 35)$.

b) Solve the linear congruence : $35x \equiv 16 \pmod{198}$

(Find a solution in the range $0 \leq x < 197$, and use it to find __all__ integer solutions.)

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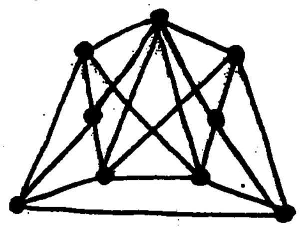
⑥ Suppose we have the following fruit supplies available:
Apples, Pears, oranges, kiwis = all unlimited
6 pomegranites ; and 5 bananas...

__Note__ fruits indistinguishable within type

How many distinguishable 7-piece fruit baskets are possible if the number of apples used must be even.

(7) Is the graph below planar or non-planar? Explain.



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(8) Consider the premises:

"It is not rainy and it is not cold."
"If it is not July, then it is rainy or cold."
"If it is July or August, then it is rainy."

Show, using appropriate rules of inference, that these premises lead to the conclusion:

"It is rainy."

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(9) There are 12 balls in an urn: 6 red, 4 blue, 2 green. 5 balls are selected at random. Find the probability that:

a.) Both green balls are among those picked.
b) An equal number of green and blue balls are picked.
c) More red balls than green balls are picked.

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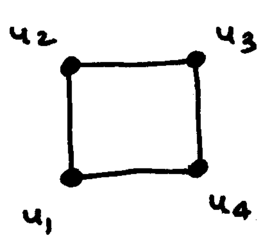
(10) Consider the 4-circuit shown below. Suppose n colors are available to color its vertices, where n is a positive integer. How many different ways are there to color its vertices so that adjacent vertices get different colors?



For example, if $n = 2$, there are two ways: