Calculus 2 Sections 1-4 Fall 2011 Exam 3 Solutions

① a) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} 2^k}{k!}$ check for abs conv : $\sum_{k=1}^{\infty} \frac{2^k}{k!}$ ratio test ...

$\lim_{k\to\infty} \frac{2^{k+1}}{(k+1)!} \frac{k!}{2^k} = \lim_{k\to\infty} \frac{2}{k+1} = 0.$ Series converges (absolutely) by ratio test.

b) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+1}$ Since $\lim_{n\to\infty} \frac{n}{n+1} = 1$, terms do not $\to 0$ as $n \to \infty$

∴ series diverges by divergence test.

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② $\sum_{k=1}^{\infty} \frac{1}{k^2+3k+2}$ a) $\sum_{k=1}^{\infty} b_k = \sum_{k=1}^{\infty} \frac{1}{k^2}$ converges by p-series ($p = 2 > 1$)

Now $\frac{1}{k^2+3k+2} < \frac{1}{k^2}$, so given series converges by comparison test.

b) $\sum_{k=1}^{n} \left(\frac{1}{k+1} - \frac{1}{k+2}\right) = S_n$

$S_1 = \frac{1}{2} - \frac{1}{3}$

$S_2 = \frac{1}{2} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \frac{1}{4}$

$S_3 = \frac{1}{2} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}} - \frac{1}{5}$

$\Rightarrow \boxed{S_n = \frac{1}{2} - \frac{1}{n+2}}$

c) $s = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \left(\frac{1}{2} - \frac{1}{n+2}\right) = \frac{1}{2}.$

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③ $\sum_{k=0}^{\infty} \frac{k^2}{3^k} (x-2)^k$ Abs ratio test : $\lim_{k\to\infty} \left| \frac{(k+1)^2}{3^{k+1}} \frac{(x-2)^{k+1}}{(x-2)^k} \cdot \frac{3^k}{k^2} \right|$

$= \lim_{k\to\infty} \left(\frac{k+1}{k}\right)^2 \frac{|x-2|}{3} = \frac{|x-2|}{3} < 1 \Rightarrow |x-2| < 3$ ← radius of conv

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④ Maclaurin Series for $f(x) = \ln(1+x)$?

$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots, \quad |x| < 1$

$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - \cdots, \quad |x| < 1$

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots + \cancel{C}^{\ln 1 = 0}, \quad |x| < 1$

So coeff of x^4 is $-\frac{1}{4}$.

Or, could compute coefficient directly via Taylor's formula :

$f(x) = \ln(1+x)$

$f'(x) = (1+x)^{-1}$

$f''(x) = -(1+x)^{-2}$

$f'''(x) = 2(1+x)^{-3}$

$f^{(4)}(x) = -6(1+x)^{-4} \Rightarrow \frac{f^{(4)}(0)}{4!} = \frac{-6}{24} = -\frac{1}{4}$ ✓

(5) a) $\displaystyle\sum_{k=2}^{\infty}\left(\frac{3}{5}\right)^k = \frac{\left(\frac{3}{5}\right)^2}{1-\frac{3}{5}} = \left(\frac{3}{5}\right)^2\left(\frac{5}{2}\right) = \frac{9}{10}$

b) $\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^k}{k^p}$ will converge conditionally if & only if $\boxed{0 < p \leq 1}$, so $p = 1, \frac{1}{2}$, etc.

c) FALSE, divergence test

d) NO, positive series which converge automatically converge absolutely.

radius of convergence is ≤ 2 ...

e) TRUE :