Discrete structures Secs 1-4 Fall 2010 Exam 3 Solutions

① $N(P_1'P_2'P_3') = N - \sum_{i=1}^{3} N(P_i) + \sum_{i \neq j} N(P_iP_j) - N(P_1P_2P_3)$

$= 256 - N(P_1) - N(P_2) - N(P_3) + N(P_1P_2) + N(P_1P_3) + N(P_2P_3) - N(P_1P_2P_3)$

$N(P_1) = \binom{8}{7} + \binom{8}{8} = 9$; $N(P_2) \Rightarrow$ 1 _ 1 _ _ _ _ _ 0 : $N(P_2) = 2^5 = 32$

$N(P_3) \Rightarrow$ _ · _ _ · _ _ · _ _ · $N(P_3) = \binom{4}{3} \cdot 2^4 = 64$

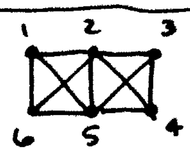
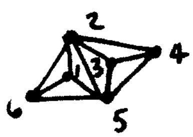
$N(P_1P_2) \Rightarrow$ 1 _ 1 _ _ _ _ _ 0 $N(P_1P_2) = 1$

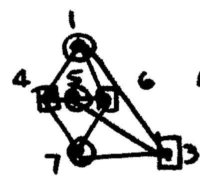
$N(P_1P_3) = \binom{4}{3} = 4$

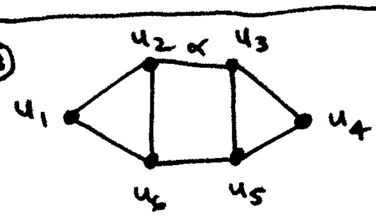
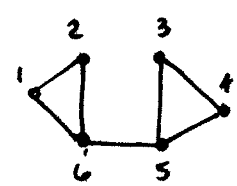
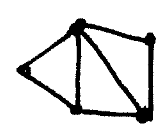
$N(P_2P_3) \Rightarrow$ 1 1 1 _ 1 1 _ 0 $N(P_2P_3) = 2^2 = 4$

$N(P_1P_2P_3) = 1$ (11111110)

So $N(P_1'P_2'P_3') = 256 - 9 - 32 - 64 + 1 + 4 + 4 - 1 = \boxed{159}$

② a)  $\cong$  planar

b)  Consider subgraph  ← K(3,3) non-planar

③  $\Rightarrow$ a)  $-$ 

So $P(G,x) = x(x-1)^3(x-2)^2 - x(x-1)(x-2)^3 = x(x-1)(x-2)^2\left[(x-1)^2 - (x-2)\right]$

$= x(x-1)(x-2)^2\left[x^2 - 3x + 3\right]$ b.) $\chi(G) = 3$ since $P(G,2) = 0$, $P(G,3) \neq 0$

④ a) In G_1, deg 2 vertex adjacent to vertices of degrees 4, 3...
In G_2, deg 2 vertex adjacent to 2 vertices of degree 3.

b) $r + v - e = 2 \Rightarrow 4 + v - 7 = 2 \Rightarrow \boxed{v = 5}$

c) No, the dual graph of any map is planar, K(3,3) is not

d) TRUE : eg,