1. (a) Use the order condition
\[ \sum_{j=0}^{k} (-j)^{m+1} \alpha_j = (m+1) \sum_{j=0}^{k} (-j)^m \beta_j, \quad m = -1, 0, \ldots \]
to obtain explicit third-order methods of the form
\[ (i) \quad w_n = w_{n-2} + h \sum_{j=1}^{3} \beta_j f_{n-j} \]
\[ (ii) \quad \sum_{j=0}^{3} \alpha_j w_{n-j} = h \beta_1 f_{n-1}, \quad \alpha_0 = 1 \]

(b) Are either of these methods stable according to the root condition? If stable, is the method weakly or strongly stable?

2. The exact solution of the IVP
\[ y' = \lambda (y - \sin t) + \cos t, \quad y(0) = 1, \quad 0 \leq t \leq 1, \]
is \( y(t) = \exp(\lambda t) + \sin t \). Consider the following second-order methods (i) explicit midpoint, (ii) AB2 and (iii) BDF2. Suppose the IVP is solved using these methods with \( h = .02 \) for the cases \( \lambda = -20 \) and \( \lambda = -80 \). Try to predict the behavior of the numerical solution by considering the region of absolute stability for each method, and then confirm it by computing the solution for each of the 6 cases. (For the two-step methods, use the starting values \( w_0 = 1 \) and \( w_1 = y(h) \).)

3. The fourth-order Adams predictor-corrector method uses the pair
Predictor step
\[ \tilde{w}_n = w_{n-1} + \frac{h}{24} [35f(t_{n-1}, w_{n-1}) - 59f(t_{n-2}, w_{n-2}) + 37f(t_{n-3}, w_{n-3}) - 9f(t_{n-4}, w_{n-4})] \]
Corrector step
\[ w_n = w_{n-1} + \frac{h}{24} [9f(t_n, \tilde{w}_n) + 19f(t_{n-1}, w_{n-1}) - 5f(t_{n-2}, w_{n-2}) + f(t_{n-3}, w_{n-3})] \]
Write a code to solve the IVP
\[ y' = f(t, y), \quad a \leq t \leq b, \quad y(a) = \gamma \]
using the predictor-corrector method for a fixed step-size \( h \) and using RK4 to obtain starting values. Use the method to solve the predator-prey system
\[
\begin{align*}
\dot{y}_1 &= y_1 - \frac{1}{2} y_1 y_2 \\
\dot{y}_2 &= -\frac{3}{4} y_2 + \frac{1}{4} y_1 y_2
\end{align*}
\]
for \( 0 \leq t \leq 15 \), with initial conditions \( y_1(0) = 3 \) and \( y_2(0) = \frac{1}{2} \). Use \( h = 15/N \), where \( N = 1500 \). Plot the two populations versus time, and compare the results with those obtained in Problem Set 2 using RK4. Print out the values \( y_1(15) \) and \( y_2(15) \) to at least 10 significant figures.