1. Use a suitable eigenfunction expansion to determine the temperature $u(r,t)$ satisfying
\[
\frac{\partial u}{\partial t} = k \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) + Q(r,t), \quad 0 \leq r \leq a, \quad t \geq 0
\]
with
\[
u(r,0) = f(r), \quad u(0,t) \text{ bounded}, \quad u(a,t) = 0
\]

2. A stretched string with linear restoring force satisfies
\[
 u_{tt} = c^2 u_{xx} - \alpha u, \quad 0 < x < \pi, \quad t > 0,
\]
where $c$ and $\alpha$ are positive constants. The string is at rest initially, i.e. $u(x,0) = u_t(x,0) = 0$, and its displacement is fixed on its right end, i.e. $u(\pi,t) = 0$. On the left, the displacement of the string is given by $u(0,t) = A \sin \omega t$, where $\omega$ is the frequency of the periodic displacement. Determine the solution in the form of an eigenfunction expansion. For what values of $\omega$ does resonance occur? (Hint: for this case it is not valid to differentiate the eigenfunction expansion term by term.)

3. Solve
\[
 u_{xx} + u_{yy} = Q(x,y), \quad 0 \leq x \leq L, \quad 0 \leq y \leq H
\]
with
\[
u_x(0,y) = u_x(L,y) = 0, \quad u(x,0) = u(x,H) = 0
\]
using (a) an expansion of the form
\[
u(x,y) = \sum_n A_n(y) \phi_n(x)
\]
where $\phi_n(x)$ are suitable one-dimensional eigenfunctions, and (b) an expansion of the form
\[
u(x,y) = \sum_\lambda C_\lambda \Phi_\lambda(x,y)
\]
where $\Phi_\lambda(x,y)$ are suitable two-dimensional eigenfunctions of a Helmholtz equation.