1. Text exercise 3.3.1, p. 113, parts (c) and (e) only. Use $L = 1$ for both parts. Which series, if any, can be differentiated term by term?

2. Let

\[ S(x) = 1 - |x - 1| \quad \text{and} \quad C(x) = x^2 \]

Find the Fourier sine series of $S(x)$ for $L = 2$ and the Fourier cosine series of $C(x)$ for $L = 1$. Sketch the function to which the series converges in both cases. Can either series be differentiated term by term? Explain briefly.

3. Consider the function $f(x) = x$ defined on the interval $0 \leq x \leq L/2$. How should $f(x)$ be extended to the interval $-L \leq x \leq L$ so that its Fourier series involves only the functions

\[ \cos \frac{\pi x}{L}, \cos \frac{3\pi x}{L}, \cos \frac{5\pi x}{L}, \ldots \]

Sketch the function to which the Fourier series converges for $-3L \leq x \leq 3L$, say. Give reasons for all answers in this problem.

4. Consider the Fourier series of $f(x)$ on the interval $-L \leq x \leq L$. Show that the Fourier coefficients satisfy Parseval’s equation

\[
\frac{1}{L} \int_{-L}^{L} f^2(x) \, dx = 2a_0^2 + \sum_{n=1}^{\infty} \left( a_n^2 + b_n^2 \right)
\]

5. Consider the wave equation with a linear forcing given by

\[ u_{tt} = c^2 u_{xx} - \alpha u, \quad 0 < x < L, \quad t > 0, \]

subject the initial conditions $u(x, 0) = f(x)$ and $u_t(x, 0) = 0$, and the boundary conditions $u(0, t) = u(L, t) = 0$. It is assumed that $c$ and $\alpha$ are positive constants.

(a) Solve for $u(x, t)$ using separation of variables.

(b) Sketch modes of vibration for the first two harmonics. Give a formula for the frequency of vibration for the first harmonic.