Math 4500 | | **Due:**
D. Schwendeman

Problem Set 2

Thursday, 2/7/08

1. Text exercise 2.3.1, p. 55, parts (b) and (d) only.

2. Text exercise 2.3.2, p. 55, parts (e) and (g) only. For part (g) you need only show graphically that there are an infinite number of eigenvalues for $\lambda > 0$ and that there are no eigenvalues for $\lambda \le 0$. What are the eigenvalues (approximately) for large positive λ?

3. Consider the heat flow in a rod with unit thermal properties and unit cross-sectional area. The temperature $u(x,t)$ solves the problem

$$u_t = u_{xx}, \qquad 0 \le x \le \pi, \quad t \ge 0,$$

with

$$u_x(0,t) = u(\pi,t) = 0 \quad \text{and} \quad u(x,0) = 3\cos\left(\frac{x}{2}\right) - \cos\left(\frac{5x}{2}\right).$$

(a) Solve for $u(x,t)$ using the method of separation of variables. (Hint: use the results of exercise 2.3.2, part (e).)

(b) Determine the total heat energy in the rod as a function of time.

(c) Determine the heat flow out of the rod at $x = \pi$ as a function time.

(d) What relationship should exist between the results of parts (b) and (c)? Verify your claim.

4. Consider the problem

$$u_{xx} + u_{yy} = 0, \qquad 0 < x < a, \quad 0 < y < b,$$

with

$$u_y(x,0) = u_y(x,b) = u(a,y) = 0, \quad \text{and} \quad u(0,y) = f(y).$$

Use separation of variables to find solutions for

$$\text{(a)} \quad f(y) = \frac{y}{b} \qquad \text{and} \qquad \text{(b)} \quad f(y) = \sin^2\left(\frac{\pi y}{b}\right).$$

Hint: consider a trigonometric identity for the square of the sine function for part (b).

5. Use separation of variables to solve Laplace's equation

$$\nabla^2 u = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0,$$

for the annular region $1 < r < 2$, $-\pi < \theta < \pi$, subject to the boundary conditions

$$u(1,\theta) = 0, \qquad u(2,\theta) = f(\theta).$$