Formula Page

Fourier series of $f(x)$ on $-L \le x \le L$:

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right)$$

where

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx, \quad a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)\,dx, \quad b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)\,dx$$

Fourier sine series of $f(x)$ on $0 \le x \le L$:

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$$

where

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)\,dx$$

Fourier cosine series of $f(x)$ on $0 \le x \le L$:

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$$

where

$$a_0 = \frac{1}{L}\int_0^L f(x)\,dx, \qquad a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)\,dx$$

Sturm-Liouville problem:

$$(p(x)\phi')' + q(x)\phi + \lambda\sigma(x)\phi = 0, \qquad a < x < b$$

$$\lambda = \frac{-p\phi\phi'|_a^b + \int_a^b (p\phi'^2 - q\phi^2)\,dx}{\int_a^b \sigma\phi^2\,dx}$$

$$\int_a^b [uL(v) - vL(u)]\,dx = p\,(uv' - vu')\big|_a^b$$

Helmholtz problem:

$$\nabla^2\phi + \lambda\phi = 0, \qquad \mathbf{x} \in R$$

$$\lambda = \frac{-\int_{\partial R} \phi\nabla\phi\cdot\mathbf{n}\,dS + \int_R |\nabla\phi|^2\,dV}{\int_R \phi^2\,dV}$$

$$\int_R \left(u\nabla^2 v - v\nabla^2 u\right)\,dV = \int_{\partial R} (u\nabla v - v\nabla u)\cdot\mathbf{n}\,dS$$

Cylindrical problems:

$$\nabla^2 u = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2}$$

$$r^2\phi'' + r\phi' + (\lambda r^2 - m^2)\phi = 0$$
$$r^2 f'' + r f' - (\alpha r^2 + m^2) f = 0$$

Spherical problems

$$\nabla^2 u = \frac{1}{\rho^2}\frac{\partial}{\partial \rho}\left(\rho^2\frac{\partial u}{\partial \rho}\right) + \frac{1}{\rho^2 \sin\phi}\frac{\partial}{\partial \phi}\left(\sin\phi\frac{\partial u}{\partial \phi}\right) + \frac{1}{\rho^2\sin^2\phi}\frac{\partial^2 u}{\partial \theta^2}$$

$$\left[\left(1 - x^2\right) g'\right]' + n(n+1)g = 0, \qquad x = \cos\phi$$

$$\left(\rho^2 R'\right)' + \left(\lambda\rho^2 - n(n+1)\right) R = 0$$

Green's functions

$$G(x,\xi) = \begin{cases} \frac{1}{c}u_1(x)u_2(\xi) & x < \xi \\ \frac{1}{c}u_1(\xi)u_2(x) & x > \xi \end{cases} \qquad Lu_i = (p(x)u_i')' + q(x)u_i = 0$$

$$G(\underline{x},\underline{\xi}) = -\sum_\lambda \left(\frac{\phi_\lambda(\underline{x})\phi_\lambda(\underline{\xi})}{\lambda N_\lambda}\right), \qquad \nabla^2\phi_\lambda + \lambda\phi_\lambda = 0$$

$$G(\underline{x},\underline{\xi}) = \frac{1}{2\pi}\ln|\underline{x} - \underline{\xi}|$$

Fourier Transforms

$$\mathcal{F}u = \frac{1}{2\pi}\int_{-\infty}^{\infty} u(x)e^{i\omega x}\,dx = \hat{u}(\omega), \qquad \mathcal{F}^{-1}\hat{u} = \int_{-\infty}^{\infty}\hat{u}(\omega)e^{-i\omega x}\,d\omega = u(x)$$

$$\mathcal{F}\left(\frac{d^n u}{dx^n}\right) = (-i\omega)^n\mathcal{F}u, \qquad \mathcal{F}^{-1}\left(e^{-\alpha\omega^2}\right) = \sqrt{\frac{\pi}{\alpha}}e^{-x^2/4\alpha}$$

$$\mathcal{F}^{-1}\left(\hat{f}(\omega)\hat{g}(\omega)\right) = \int_{-\infty}^{\infty} f(\xi)g(x-\xi)\,d\xi = \int_{-\infty}^{\infty} f(x-\xi)g(\xi)\,d\xi$$

Fourier Sine and Cosine Transforms

$$\mathcal{S}u = \frac{2}{\pi}\int_0^{\infty} u(x)\sin\omega x\,dx = \hat{u}(\omega), \qquad \mathcal{S}^{-1}\hat{u} = \int_0^{\infty}\hat{u}(\omega)\sin\omega x\,d\omega = u(x)$$

$$\mathcal{C}u = \frac{2}{\pi}\int_0^{\infty} u(x)\cos\omega x\,dx = \hat{u}(\omega), \qquad \mathcal{C}^{-1}\hat{u} = \int_0^{\infty}\hat{u}(\omega)\cos\omega x\,d\omega = u(x)$$

$$\mathcal{S}u_{xx} = \frac{2\omega}{\pi}u(0) - \omega^2\mathcal{S}u, \qquad \mathcal{C}u_{xx} = -\frac{2}{\pi}u_x(0) - \omega^2\mathcal{C}u$$