110.) $\dot{x} = x - x^2 - xy \quad = F(x,y)$

$\dot{y} = 3y - xy - 2y^2 \quad = G(x,y)$

Equilibria: $\quad x - x^2 - xy = 0$

$3y - xy - 2y^2 = 0$

$x(1 - x - y) = 0$

$y(3 - x - 2y) = 0$

4 cases: (1) $x = 0 \quad y = 0$

(2) $x = 0 \quad y = \frac{3}{2}$

(3) $x = 1 \quad y = 0$

(4) $\left.\begin{array}{l} x + y = 1 \\ x + 2y = 3 \end{array}\right\} \quad y = 2, \quad x = -1$

Linearization:

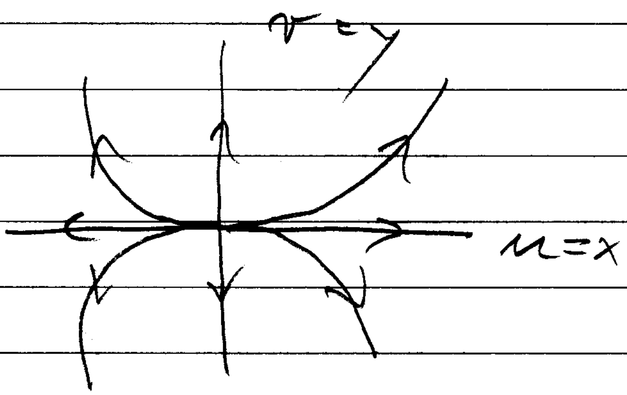
$$A(x,y) = \begin{bmatrix} F_x(x,y) & F_y(x,y) \\ G_x(x,y) & G_y(x,y) \end{bmatrix} =$$

$$= \begin{bmatrix} 1 - 2x - y & -x \\ -y & 3 - x - 4y \end{bmatrix}$$

(1) $x = y = 0$ $\quad u = x, \; v = y$

$$\begin{pmatrix} u \\ v \end{pmatrix}' = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$r_1 = 1, \; \xi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \quad r_2 = 3, \; \xi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

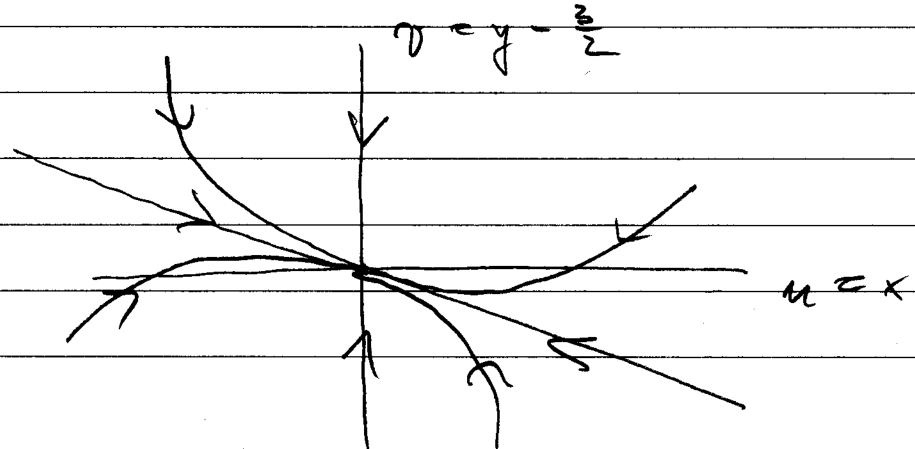


(0,0) IS A SOURCE, UNSTABLE

(2) $x = 0, \; y = 3/2$ $\quad u = x, \; v = y - \frac{3}{2}$

$$\begin{pmatrix} u \\ v \end{pmatrix}' = \begin{bmatrix} -\frac{1}{2} & 0 \\ -\frac{3}{2} & -3 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$r_1 = -\frac{1}{2}, \; \xi^{(1)} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}, \quad r_2 = -3, \; \xi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$



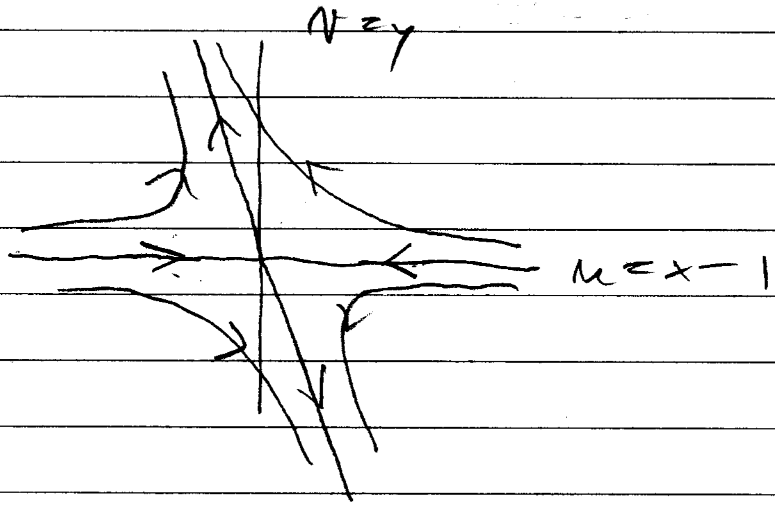
$(0, \frac{3}{2})$ is A SINK, ASYMPTOTICALLY STABLE

13) $x = 1, \; y = 0 \qquad u = x - 1 \qquad v = y$

$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{bmatrix} -2 & -1 \\ 0 & 2 \end{bmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

$r_1 = -2 \quad z^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad r_2 = 2 \quad z^{(2)} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$

$(1, 0)$ IS A SADDLE, UNSTABLE



14) $x = -1, \; y = 2 \qquad u = x + 1 \qquad v = x - 2$

$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{bmatrix} 1 & 1 \\ -2 & -4 \end{bmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

$$\begin{vmatrix} 1-r & 1 \\ -2 & -4-r \end{vmatrix} = (r-1)(r+4) + 2 =$$

$$= r^2 + 3r - 2 = 0$$

$$r_{1,2} = -\frac{3}{2} \pm \sqrt{\frac{9}{4} + 2} = -\frac{3}{2} \pm \frac{\sqrt{17}}{2}$$

Since $\sqrt{17} \approx 4$ $\Rightarrow$ $r_1 > 0$

$r_2 < 0$

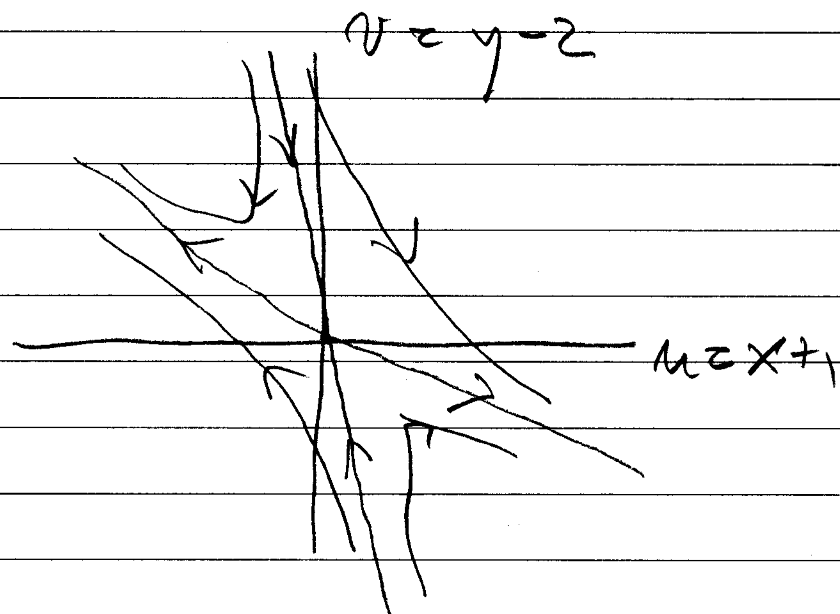
<u>Eigenvectors</u>: $r_1 = -\frac{3}{2} + \frac{\sqrt{17}}{2}$

$$\begin{bmatrix} \frac{5}{2} - \frac{\sqrt{17}}{2} & 1 \\ -2 & -\frac{5}{2} - \frac{\sqrt{17}}{2} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = 0 \qquad z^{(1)} = \begin{bmatrix} 1 \\ -\frac{5}{2} + \frac{\sqrt{17}}{2} \end{bmatrix}$$

$r_2 = -\frac{3}{2} - \frac{\sqrt{17}}{2}$

$$\begin{bmatrix} \frac{5}{2} + \frac{\sqrt{17}}{2} & 1 \\ -2 & -\frac{5}{2} + \frac{\sqrt{17}}{2} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = 0 \qquad z^{(2)} = \begin{bmatrix} 1 \\ -\frac{5}{2} - \frac{\sqrt{17}}{2} \end{bmatrix}$$

$$z^{(1)} \sim \begin{bmatrix} 1 \\ -\frac{1}{2} \end{bmatrix} \qquad z^{(2)} \sim \begin{bmatrix} 1 \\ -\frac{9}{2} \end{bmatrix}$$



(-1, 2) is A SADDLE

UNSTABLE

III.) $\dot{x} = 1 - y \qquad \dot{y} = x^2 - y^2$

Equilibria: $\left.\begin{array}{l} 1 - y = 0 \\ x^2 - y^2 = 0 \end{array}\right\} \quad \begin{array}{l} y = 1 \\ x = \pm 1 \end{array}$

Linearization

$$A(x,y) = \begin{bmatrix} 0 & -1 \\ 2x & -2y \end{bmatrix}$$

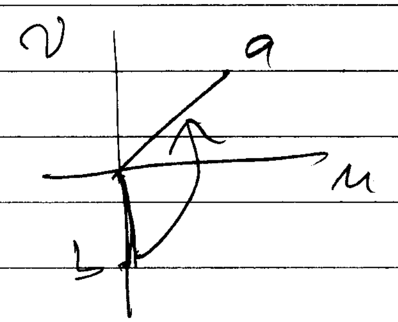
(1,1) : $\quad u = x - 1 \qquad v = y - 1$

$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{bmatrix} 0 & -1 \\ 2 & -2 \end{bmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

$$\begin{vmatrix} -r & -1 \\ 2 & -2-r \end{vmatrix} = r(2+r) + 2 = r^2 + 2r + 2 = 0$$

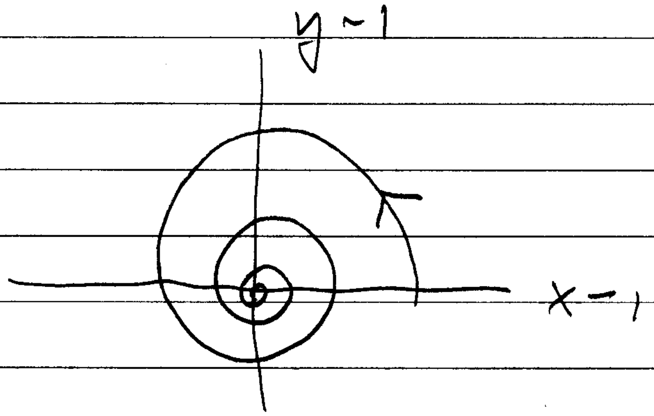
$$r = -1 \pm \sqrt{1-2} = -1 \pm i$$

$$(1-i) z_1 - z_2 = 0 \Rightarrow z = \begin{pmatrix} 1 \\ 1-i \end{pmatrix} =$$



$$= \underbrace{\begin{pmatrix} 1 \\ 1 \end{pmatrix}}_{a} + i \underbrace{\begin{pmatrix} 0 \\ -1 \end{pmatrix}}_{b}$$

(1, 1)
IS A
SPIRAL SINK
ASYMPTOTICALLY
STABLE



$(-1, 1)$: $u = x + 1$ $v = x - 1$

$$\begin{pmatrix}\dot u\\ \dot v\end{pmatrix} = \begin{bmatrix}0 & -1\\ -2 & -2\end{bmatrix}\begin{bmatrix}u\\ v\end{bmatrix}$$

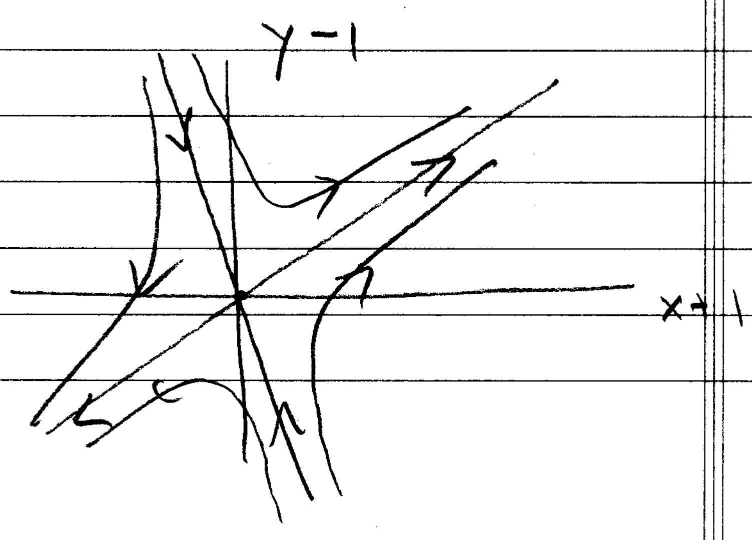
$$\begin{vmatrix}-r & -1\\ -2 & -2-r\end{vmatrix} = r(r+2) - 2 = r^2 + 2r - 2 = 0$$

$$r_{1,2} = -1 \pm \sqrt{1+2} = -1 \pm \sqrt{3}$$

$r_1 = -1 + \sqrt{3} > 0$: $z^{(1)} = \begin{pmatrix}1\\ -1+\sqrt{3}\end{pmatrix} \sim \begin{pmatrix}1\\ 0.7\end{pmatrix}$

$r_2 = -1 - \sqrt{3} < 0$: $z^{(2)} = \begin{pmatrix}1\\ -1-\sqrt{3}\end{pmatrix} \sim \begin{pmatrix}1\\ -2.7\end{pmatrix}$

(-1, 1) IS
A SADDLE,
UNSTABLE



112. $\dot{x} = x\left(\frac{3}{2} - x - \frac{1}{2}y\right)$

$\dot{y} = y\left(2 - y - \frac{3}{4}x\right)$

Equilibria

1.) $x = 0 \quad y = 0$

2.) $x = 0 \quad y = 2$

3.) $x = \frac{3}{2} \quad y = 0$

4.) $\left.\begin{aligned} 2x + y &= 3 \\ 3x + 4y &= 8 \end{aligned}\right\}$ $\quad y = 3 - 2x$

$3x + 12 - 8x = 8$

$5x = 4$

$x = \frac{4}{5}, \; y = \frac{7}{5}$ $\quad y = 3 - \frac{8}{5} = \frac{7}{5}$

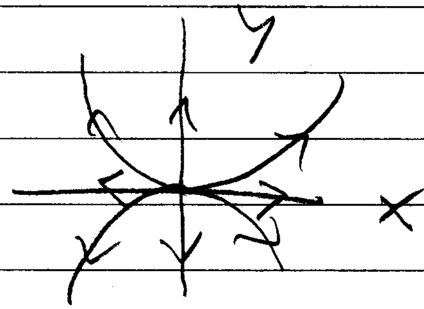
Linearization:

$$A = \begin{bmatrix} \frac{3}{2} - 2x - \frac{1}{2}y & -\frac{1}{2}x \\ -\frac{3}{4}y & 2 - 2y - \frac{3}{4}x \end{bmatrix}$$

$\underline{(0,0)}$: $u = x$ $v = y$

$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{bmatrix} \frac{3}{2} & 0 \\ 0 & 2 \end{bmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

$r_1 = \frac{3}{2}$, $\xi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$; $r_2 = 2$, $\xi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

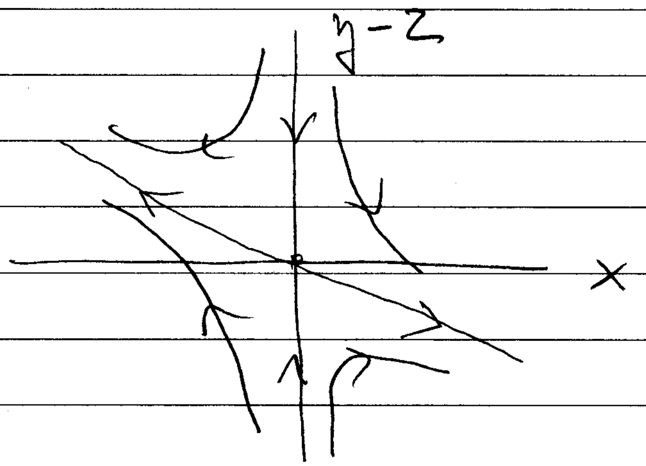


(0,0) is A SOURCE, UNSTABLE

$\underline{(0,2)}$: $u = x$ $v = y-2$

$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{3}{2} & -2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$r_1 = \frac{1}{2}$ $\xi^{(1)} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$, $r_2 = -2$ $\xi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$



(0,2) is A SADDLE, UNSTABLE

(I)

$\left(\frac{3}{2}, 0\right)$: $u = x - \frac{3}{2}$, $v = y$

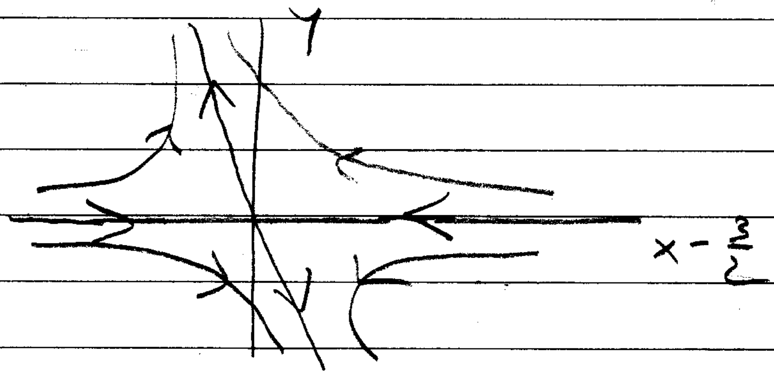
$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{bmatrix} -\frac{3}{2} & -\frac{3}{4} \\ 0 & \frac{7}{8} \end{bmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

$r_1 = -\frac{3}{2}$, $z^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$; $r_2 = \frac{7}{8}$, $z^{(2)} = \begin{pmatrix} 3 \\ -19 \end{pmatrix}$

$\left(\frac{3}{2}, 0\right)$ IS

A SADDLE,

UNSTABLE



$\left(\frac{4}{5}, \frac{7}{5}\right)$: $u = x - \frac{4}{5}$ $v = y - \frac{7}{5}$

$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{bmatrix} \frac{3}{2} - \frac{8}{5} - \frac{7}{10} & -\frac{2}{5} \\ -\frac{21}{20} & 2 - \frac{14}{5} - \frac{3}{5} \end{bmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{bmatrix} -\frac{4}{5} & -\frac{2}{5} \\ -\frac{21}{20} & -\frac{7}{5} \end{bmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

$$\begin{vmatrix} -r-\frac{4}{5} & -\frac{2}{5} \\ -\frac{21}{20} & -r-\frac{7}{5} \end{vmatrix} = \left(r+\frac{4}{5}\right)\left(r+\frac{7}{5}\right) - \frac{42}{100}$$

$$= r^2 + \frac{11}{5}r + \frac{28}{25} - \frac{42}{100} =$$

$$= r^2 + \frac{11}{5} + \frac{7}{10} = 0$$

$$r_{1,2} = -\frac{11}{10} \pm \sqrt{\frac{121}{100} - \frac{7}{10}} = -\frac{11}{10} \pm \frac{\sqrt{51}}{10}$$

Both < 0

<u>Eigenvectors</u>

$$r_1 = -\frac{11}{10} + \frac{\sqrt{51}}{10}$$

$$\left(\frac{3}{10} - \frac{\sqrt{51}}{10}\right) z_1 - \frac{2}{5} z_2 = 0$$

$$z^{(1)} = \begin{pmatrix} 4 \\ 3-\sqrt{51} \end{pmatrix} \sim \begin{pmatrix} 4 \\ -4 \end{pmatrix}$$

$$r_2 = -\frac{11}{10} - \frac{\sqrt{51}}{10}$$

$$\left(\frac{3}{10} + \frac{\sqrt{51}}{10}\right) z_1 - \frac{2}{5} z_2 = 0$$

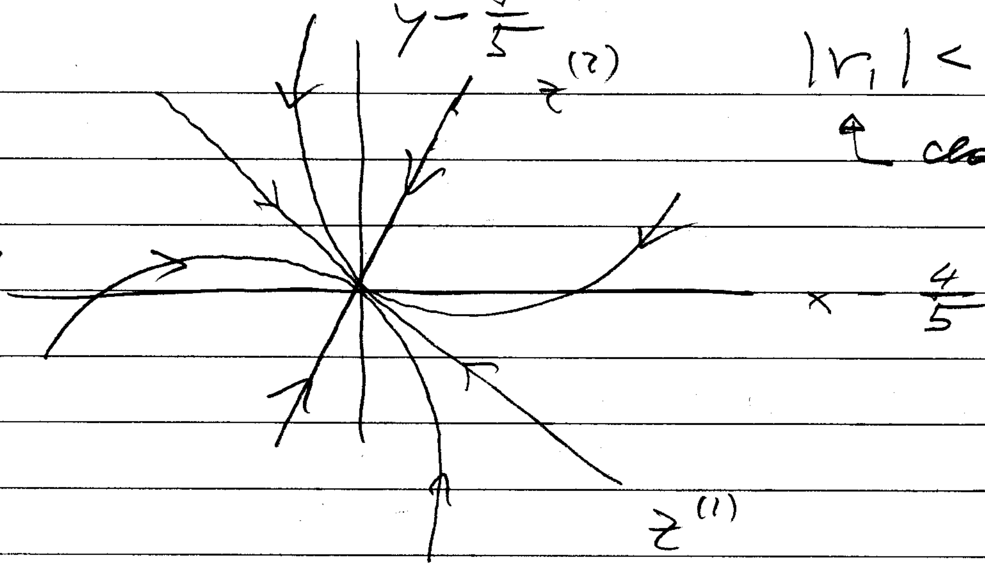
$$z^{(2)} = \begin{pmatrix} 4 \\ 3+\sqrt{51} \end{pmatrix} \sim \begin{pmatrix} 4 \\ 10 \end{pmatrix}$$

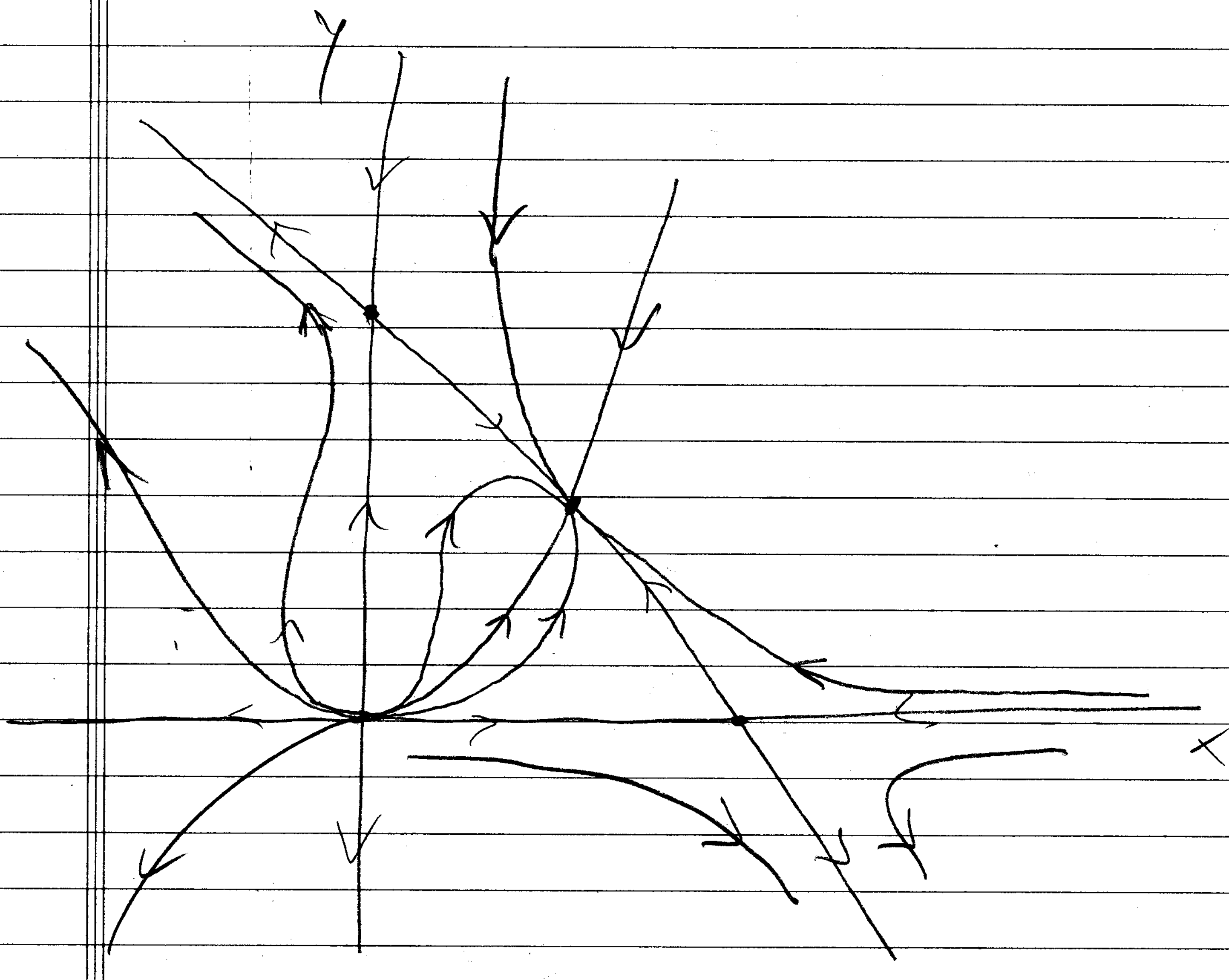
6

$\left(\frac{4}{5}, \frac{7}{5}\right)$ IS A SINK, ASYMPTOTICALLY STABLE

$|r_1| < |r_2|$

closer to 0





113.) $\dot{x} = x\left(\frac{3}{2} - x - \frac{1}{2}y\right)$

$\dot{y} = y\left(2 - \frac{1}{2}y - \frac{3}{2}x\right)$

EQUILIBRIA 1.) $x = y = 0$

2.) $x = 0, \quad y = 4$

3.) $x = \frac{3}{2}, \quad y = 0$

4.) $\left.\begin{array}{l} 2x + y = 3 \\ 3x + y = 4 \end{array}\right\} \quad x = 1, \quad y = 1$

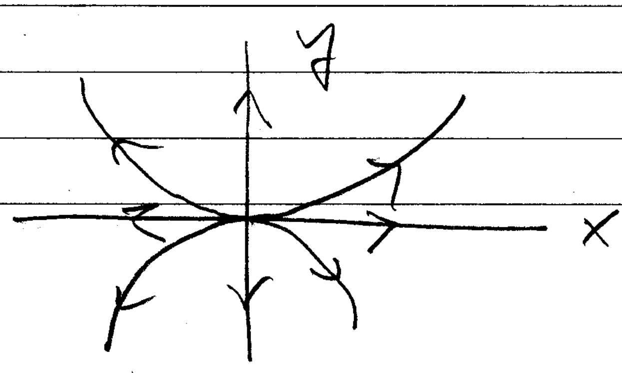
LINEARIZATION $A(x,y) = \begin{bmatrix} \frac{3}{2} - 2x - \frac{1}{2}y & -\frac{1}{2}x \\ -\frac{3}{2}y & 2 - y - \frac{3}{2}x \end{bmatrix}$

(0,0): $u = x \quad v = y$

$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{bmatrix} \frac{3}{2} & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$r_1 = \frac{3}{2} \quad \xi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad r_2 = 2 \quad \xi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

(0,0) IS A SOURCE, UNSTABLE

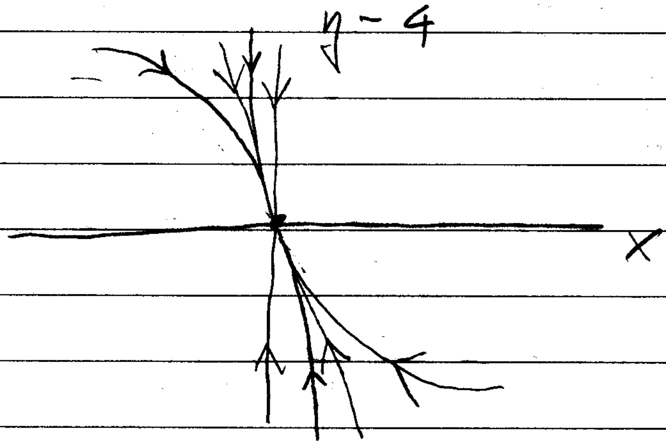


$(0,4)$: $\quad u = x \quad v = y-4$

$$\begin{pmatrix} \dot u \\ \dot v \end{pmatrix} = \begin{bmatrix} -\frac{1}{2} & 0 \\ -6 & -2 \end{bmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

$$r_1 = -\tfrac{1}{2} \quad z^{(1)} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}, \quad r_2 = -2 \quad z^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$(0,4)$ IS A SINK, ASYMPTOTICALLY STABLE

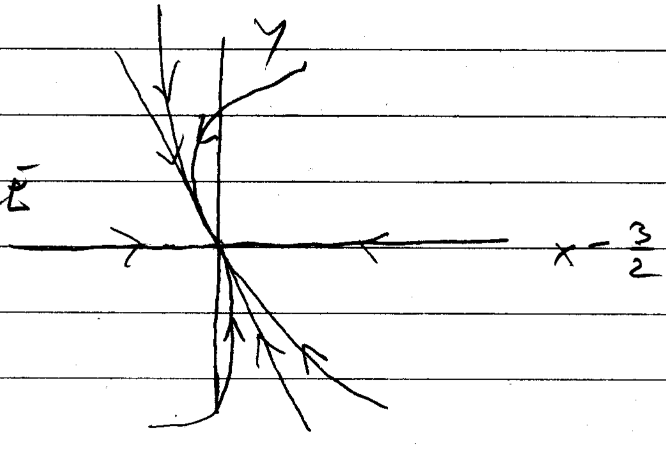


$(\frac{3}{2}, 0)$: $\quad u = x - \frac{3}{2} \quad v = y$

$$\begin{pmatrix} \dot u \\ \dot v \end{pmatrix} = \begin{bmatrix} -\frac{3}{2} & -\frac{3}{4} \\ 0 & -\frac{1}{4} \end{bmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

$$r_1 = -\tfrac{3}{2}, \quad z^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \quad r_2 = -\tfrac{1}{4}, \quad z^{(2)} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$$

$(\frac{3}{2}, 0)$ IS A SINK, ASYMPTOTICALLY STABLE



$\underline{(1,1)}:\quad u = x-1 \qquad v = y-1$

$$\begin{pmatrix} \dot u \\ \dot v \end{pmatrix} = \begin{bmatrix} -1 & -\frac{1}{2} \\ -\frac{3}{2} & -\frac{1}{2} \end{bmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

$$\begin{vmatrix} -1-r & -\frac{1}{2} \\ -\frac{3}{2} & -\frac{1}{2}-r \end{vmatrix} = (1+r)\left(\tfrac{1}{2}+r\right) - \frac{3}{4} =$$

$$= r^2 + \frac{3}{2}r - \frac{1}{4} = 0$$

$$r_{1,2} = -\frac{3}{4} \pm \sqrt{\frac{9}{16}+\frac{1}{4}} = -\frac{3}{4} \pm \frac{\sqrt{13}}{4}$$

$$r_1 = -\frac{3}{4} + \frac{\sqrt{13}}{4} \sim \frac{1}{4} > 0$$

$$\underline{r_2 < 0}$$

$$r_1 = -\frac{3}{4} + \frac{\sqrt{13}}{4} \qquad \left(-\frac{1}{4} - \frac{\sqrt{13}}{4}\right) z_1 - \frac{1}{2} z_2 = 0$$

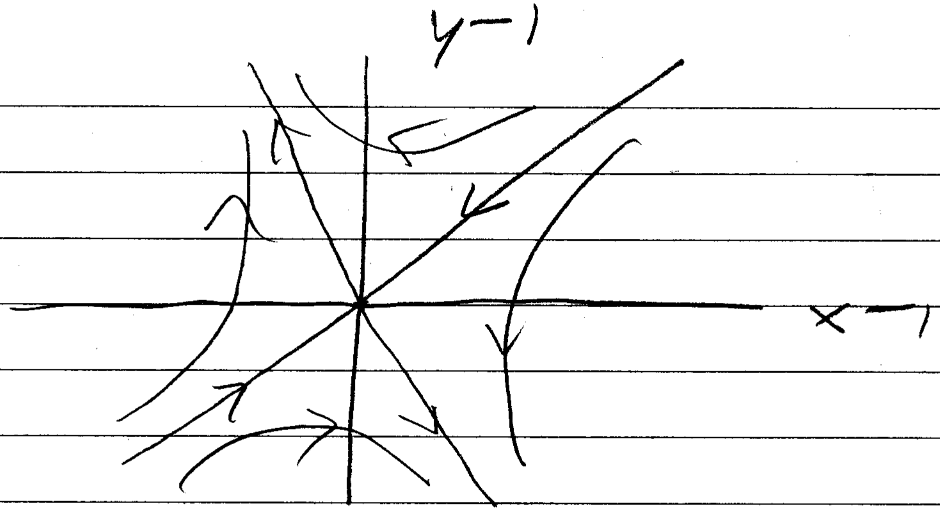
$$z_1 = 2, \quad z_2 = -(1+\sqrt{13})$$

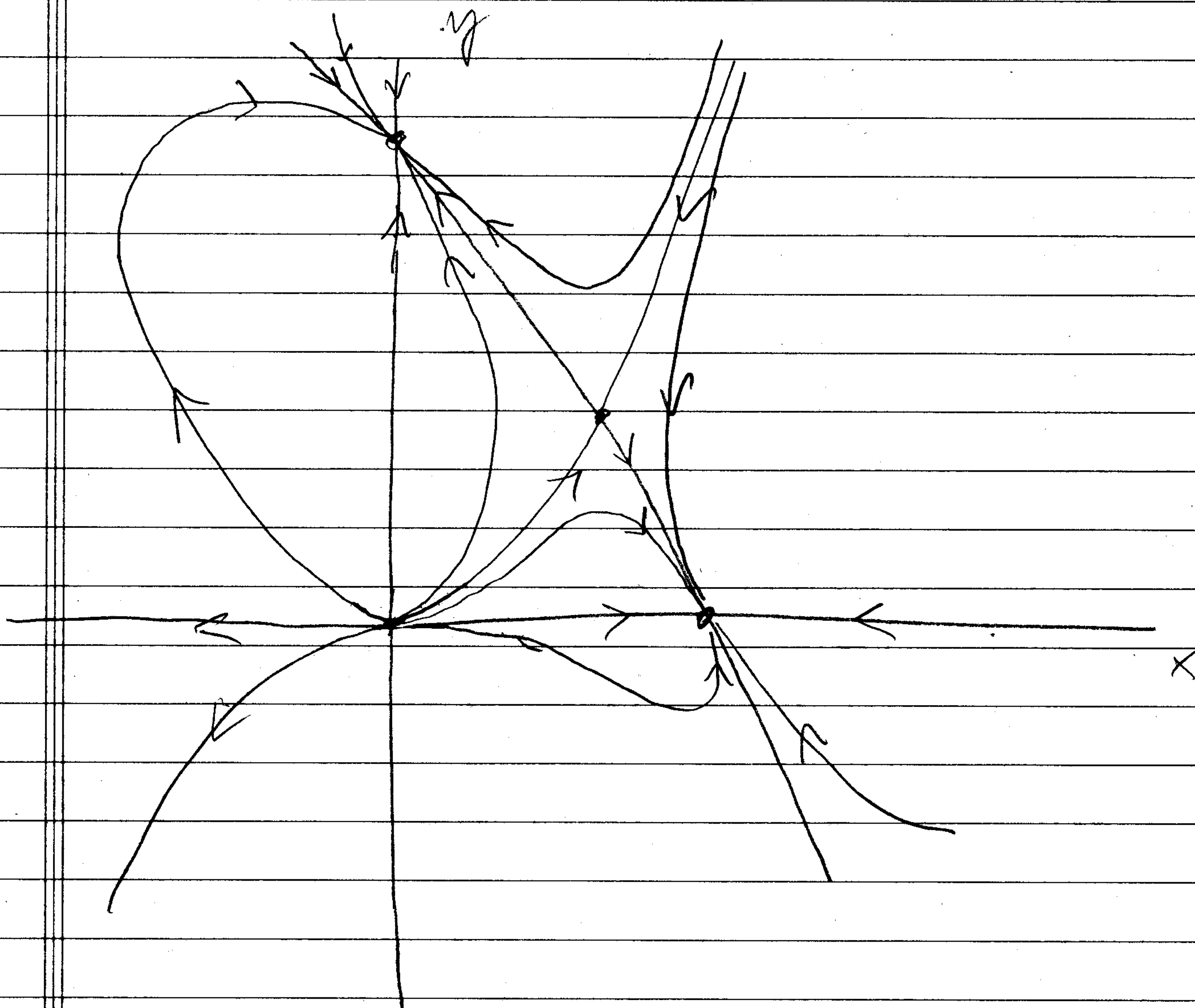
$$z^{(1)} = \begin{pmatrix} 2 \\ -(1+\sqrt{13}) \end{pmatrix} \sim \begin{pmatrix} 2 \\ -5 \end{pmatrix}$$

$$r_2 = -\frac{3}{4} - \frac{\sqrt{13}}{4} \qquad z^{(2)} = \begin{pmatrix} 2 \\ -1+\sqrt{13} \end{pmatrix} \sim \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

(1,1) IS A
SADDLE

UNSTABLE





114.) $\dot{x} = x(1-x-y)$

$\dot{y} = y\left(\frac{3}{2}-y-x\right)$

EQUILIBRIA 1.) $x=y=0$

2.) $x=0 \quad y=\frac{3}{2}$

3.) $x=1 \quad y=0$

4.) $\left.\begin{array}{l} x+y=1 \\ x+y=\frac{3}{2} \end{array}\right\}$ NO SOLUTION!

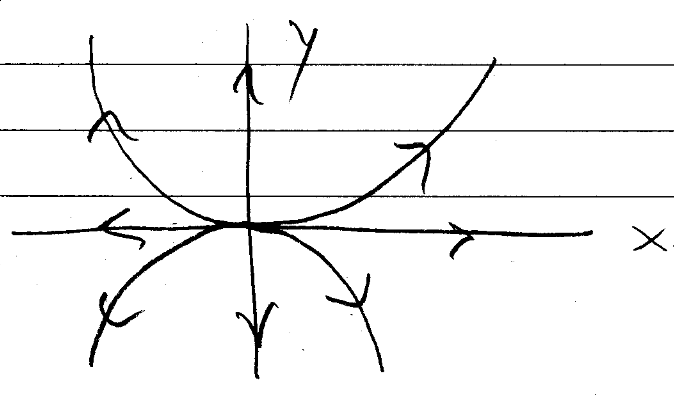
LINEARIZATION: $A = \begin{bmatrix} 1-2x-y & -x \\ -y & \frac{3}{2}-2y-x \end{bmatrix}$

(0,0) $\quad u=x \quad v=y$

$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{3}{2} \end{bmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

$r_1=1 \quad z^{(1)}=\begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad r_2=\frac{3}{2} \quad z^{(2)}=\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

(0,0) IS A SOURCE, UNSTABLE



$(0,\frac{3}{2})$: $u = x$ $\quad v = y - \frac{3}{2}$

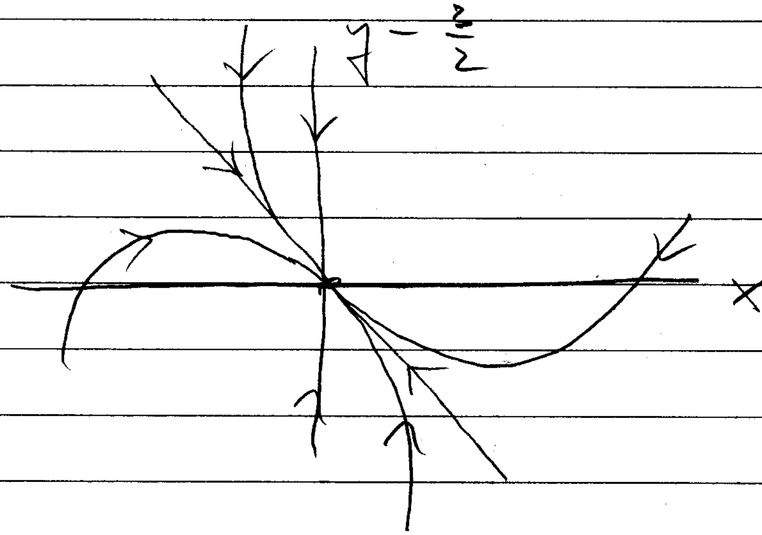
$$\begin{pmatrix} \dot u \\ \dot v \end{pmatrix} = \begin{bmatrix} -\frac{1}{2} & 0 \\ -\frac{3}{2} & -\frac{3}{2} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$r_1 = -\frac{1}{2} \quad \xi^{(1)} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}, \quad r_2 = -\frac{3}{2} \quad \xi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$(0,\frac{3}{2})$ is

A SINK

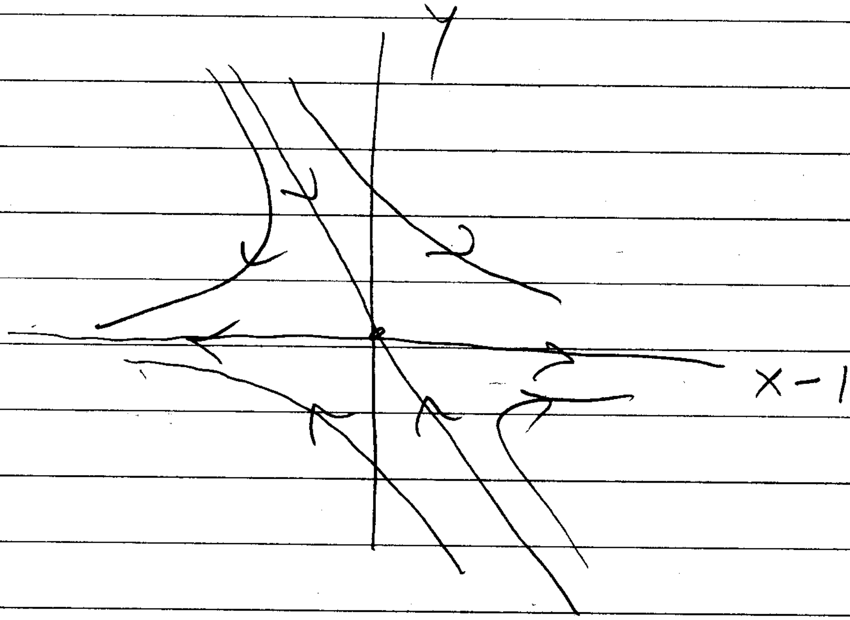
ASYMPTOTICALLY STABLE

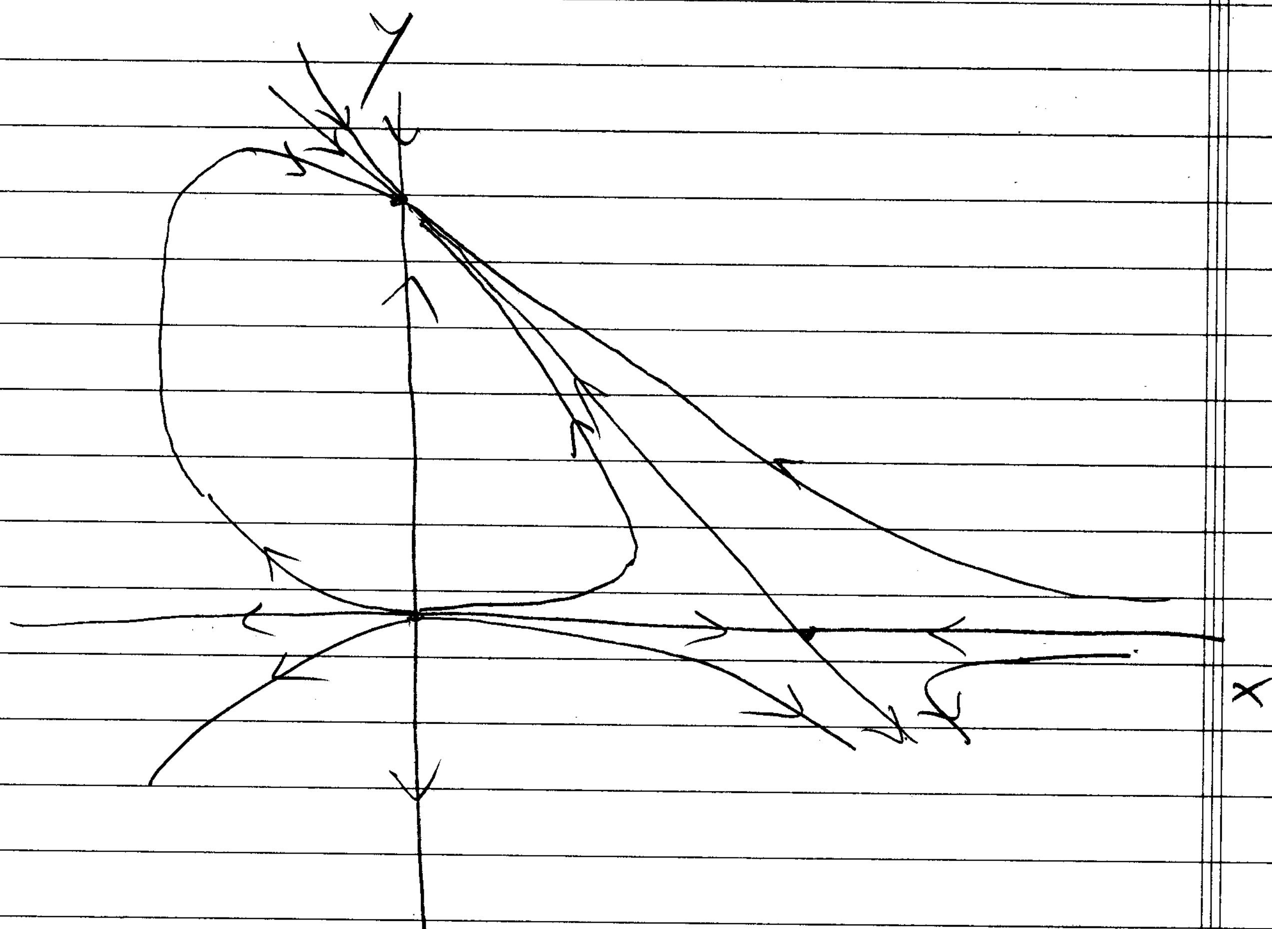


$(1,0)$ $\quad u = x - 1$ $\quad v = y$

$$\begin{pmatrix} \dot u \\ \dot v \end{pmatrix} = \begin{bmatrix} -1 & -1 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$r_1 = -1 \quad \xi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad r_2 = \frac{1}{2} \quad \xi^{(2)} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$


y
x - 1


y
x

115.) $\dot{x} = x\left(1 - x - \frac{1}{2}y\right)$

$\dot{y} = y\left(\frac{5}{2} - \frac{3}{2}y - \frac{1}{4}x\right)$

EQUILIBRIA: 1.) $x = y = 0$

2.) $x = 1,\ y = 0$

3.) $x = 0,\ y = \frac{5}{3}$

4.) $2x + y = 2 \Rightarrow y = 2 - 2x$

$x + 6y = 10$ $\quad x + 12 - 12x = 10$

$2 = 11x$

$$\left.\begin{aligned} x &= \frac{2}{11} \\ y &= \frac{18}{11} \end{aligned}\right\}$$

LINEARIZATION

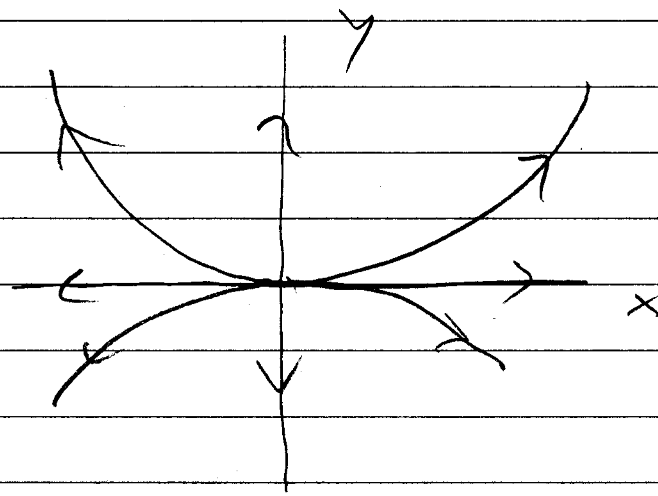
$$A = \begin{bmatrix} 1 - 2x - \frac{1}{2}y & -\frac{1}{2}x \\ -\frac{1}{4}y & \frac{5}{2} - 3y - \frac{1}{4}x \end{bmatrix}$$

$(0,0)$: $u = x$ $v = y$

$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{5}{2} \end{bmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

$r_1 = 1$ $\xi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $r_2 = \frac{5}{2}$ $\xi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$(0,0)$ IS A SOURCE, UNSTABLE



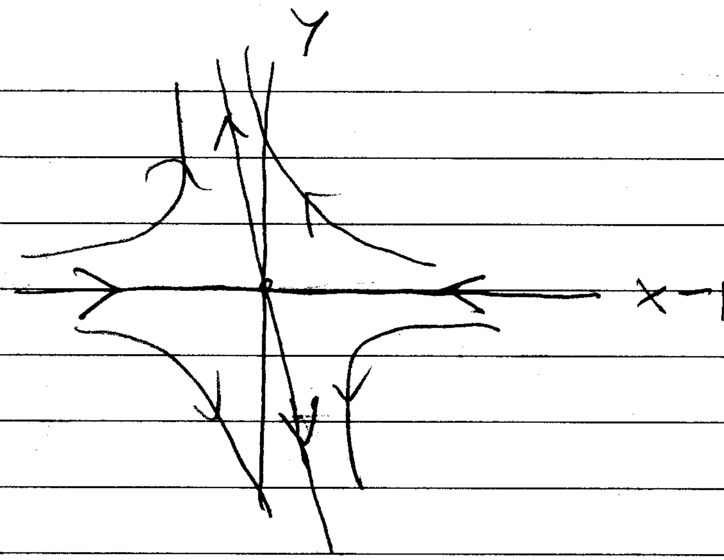
$(1,0)$ $u = x - 1$ $v = y$

$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{bmatrix} -1 & -\frac{1}{2} \\ 0 & \frac{9}{4} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$r_1 = -1$ $\xi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $r_2 = \frac{9}{4}$ $\xi^{(2)} = \begin{pmatrix} 2 \\ -13 \end{pmatrix}$

(11)

$(1,0)$ IS A SADDLE, UNSTABLE



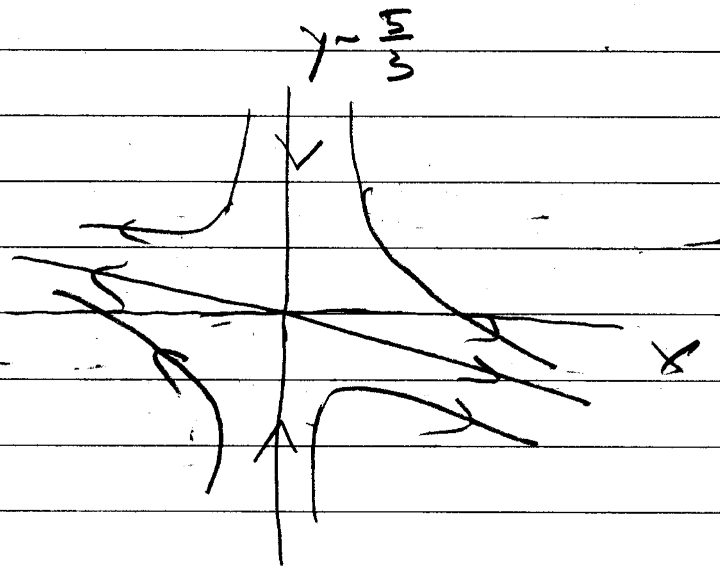
$\underline{\left(0, \frac{5}{3}\right)}$: $u = x \qquad v = y - \frac{5}{3}$

$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ -\frac{5}{12} & -\frac{5}{2} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$r_1 = \frac{1}{6} \qquad z^{(1)} = \begin{pmatrix} 32 \\ -5 \end{pmatrix}, \qquad r_2 = -\frac{5}{2} \quad z^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$\left(0, \frac{5}{3}\right)$ IS A SADDLE

UNSTABLE



$$\underline{\left(\frac{2}{11}, \frac{18}{11}\right)} \qquad u = x - \frac{2}{11} \qquad v = y - \frac{18}{11}$$

$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{bmatrix} 1 - \frac{4}{11} - \frac{9}{11} & -\frac{1}{11} \\ -\frac{9}{22} & \frac{5}{2} - \frac{54}{11} - \frac{1}{22} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{bmatrix} -\frac{2}{11} & -\frac{1}{11} \\ -\frac{9}{22} & -\frac{54}{22} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$\begin{vmatrix} -r - \frac{2}{11} & -\frac{1}{11} \\ -\frac{9}{22} & -r - \frac{54}{22} \end{vmatrix} = \left(r + \frac{2}{11}\right)\left(r + \frac{54}{22}\right) - \frac{9}{242}$$

$$= r^2 + \frac{58}{22} r + \frac{99}{242} =$$

$$= r^2 + \frac{29}{11} r + \frac{9}{22} = 0$$

$$r = -\frac{29}{22} \pm \sqrt{\left(\frac{29}{22}\right)^2 - \frac{9}{22}} =$$

$$= -\frac{29}{22} \pm \frac{\sqrt{643}}{22} = -\frac{29}{22} \pm \frac{25.36\ldots}{22}$$

<u>BOTH < 0 :</u>

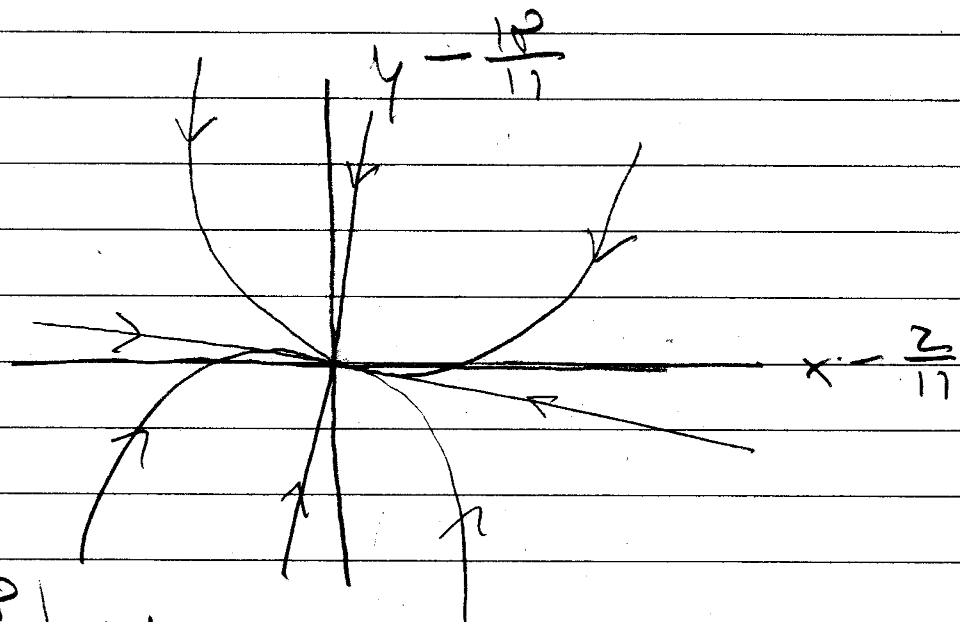
EIGENVECTORS (DO BOTH AT THE SAME TIME :)

$$\left(\frac{25}{22} \mp \frac{\sqrt{693}}{22}\right) z_1 - \frac{z_2}{11} = 0$$

$$(25 \mp 25.36\ldots) z_1 - 2z_2 = 0$$

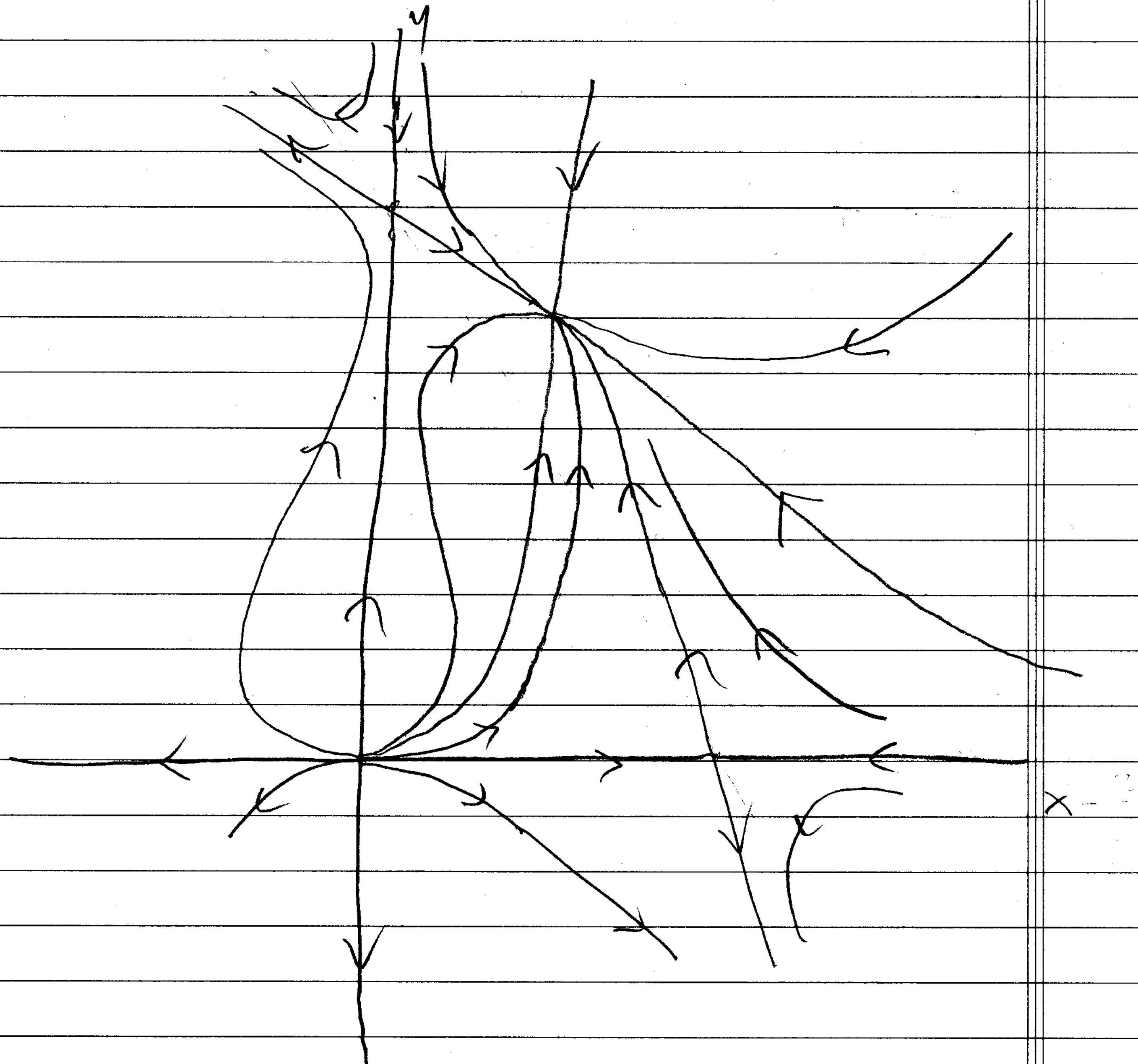
$$z^{(1)} = \begin{pmatrix} 2 \\ -0.36\ldots \end{pmatrix}$$

$$z^{(2)} \sim \begin{pmatrix} 2 \\ 50 \end{pmatrix}$$



$\left(\frac{2}{11}, \frac{10}{11}\right)$ IS

A SINK, ASYMPTOTICALLY STABLE


y
x