Outline

9. antenna theory
10. spotlight SAR
11. stripmap SAR
   (a) construction of imaging operator
   (b) a shortcoming of the continuum model
   (c) resolution
   (d) analysis of singularities
12. understanding SAR images
13. the state of the art
   (a) SAR interferometry
14. current research and open problems
Books

• B. Borden, Radar Imaging of Airborne Targets, Institute of Physics, 1999.


• I.G. Cumming and F.H. Wong, Digital Processing of SAR Data: Algorithms and Implementation, Artech House, 2005
The Incident Wave

The field from the antenna is $\mathcal{E}^{in}$, which satisfies

$$(\nabla^2 - c^{-2}\partial_t^2)\mathcal{E}^{in}(t, \mathbi{x}) = j(t, \mathbi{x})$$

$$\Rightarrow$$

$$\mathcal{E}^{in}(t, \mathbi{x}) = \int_{\text{antenna}} \int \frac{e^{-i\omega(t-t' - |\mathbi{x} - \mathbi{y}|/c)}}{8\pi^2|\mathbi{x} - \mathbi{y}|} j(t', \mathbi{y}) \, d\omega dt' d\mathbi{y}$$

$$= \int_{\text{antenna}} \int \frac{e^{-i\omega(t - |\mathbi{x} - \mathbi{y}|/c)}}{8\pi^2|\mathbi{x} - \mathbi{y}|} J(\omega, \mathbi{y}) \, d\omega d\mathbi{y}$$

where $j =$ Fourier transform of $J$.

This model allows for:

- arbitrary waveforms, spatially distributed antennas
- array antennas in which different elements are activated with
different waveforms
Let \( y = \gamma(s) + q \), use far-field expansion: 
\[
|q| << |R_{s,x}| \Rightarrow 
\]
\[
|x - y| = |R_{s,x} - q| = |R_{s,x}| - \hat{R}_{s,x} \cdot q + O(|R_{s,x}|^{-1})
\]
\[
\Rightarrow \mathcal{E}^{in}(t, x) \approx \int \frac{e^{-i\omega(t - |R_{s,x}|/c)}}{8\pi^2|R_{s,x}|} \int_{\text{antenna}} e^{-ik\hat{R}_{s,x} \cdot q} j(\omega, q) \, dq \, d\omega 
\]
\[
\mathcal{F}[J](\omega, s, x)
\]
Antenna beam pattern at each frequency is Fourier transform of (effective) current density on antenna
Putting it all together ...

- Born approximation for $\mathcal{E}^{sc}$:

$$\mathcal{E}^{sc}(t, y) = \int \frac{e^{-i\omega(t-\tau-|y-x|/c)}}{8\pi^2 |y-x|} d\omega V(x) \partial^2_{\tau} \mathcal{E}^{in}(\tau, s, x) d\tau d\omega$$

- Incident field is

$$\mathcal{E}^{in}(\tau, x) \approx \int \frac{e^{-i\omega'(|\mathbf{R}_{s,x}|/c)}}{8\pi^2 |\mathbf{R}_{s,x}|} \mathcal{F}[J](\omega', s, x) d\omega'$$

- Plug this in, use $\int e^{i(\omega-\omega')\tau} d\tau = 2\pi \delta(\omega-\omega') \Rightarrow$

$$\mathcal{E}^{sc}(t, y) = \int \int \frac{e^{-i\omega(t-|y-x|/c-|\mathbf{R}_{s,x}|/c)}}{8\pi^2 |y-x| 4\pi |\mathbf{R}_{s,x}|} \mathcal{F}[J](\omega, s, x) d\omega V(x) dx$$

- Use antenna reception model:

$$s_{rec}(t, s) = \int \int_{\text{antenna}} \mathcal{E}^{sc}(t-t', \gamma(s)-y) \mathcal{W}(t', y) dy dt'$$

$$= \int \int \frac{e^{-i\omega(t-2|R_{s,x}|/c)}}{32\pi^3 |R_{s,x}|^2} \mathcal{F}[J](\omega, s, x) \mathcal{F}[W](\omega, s, x) d\omega V(x) dx$$
Apply matched filter

output of correlation receiver is of the form

\[
d(t, s) = \int \int e^{-i\omega (t-2|\mathbf{R}_s, \mathbf{x}|/c)} A(\omega, s, \mathbf{x}) d\omega V(\mathbf{x}) d\mathbf{x}
\]

A includes factors for:
1. geometrical spreading
2. antenna beam patterns
3. waveform sent to antenna
data is of the form

\[ d(t, s) = \iiint e^{-i\omega(t - 2|R_s,x|/c)} A(\omega, s, x) d\omega V(x) dx \]

from \( d \), want to reconstruct \( V \).
Fourier transform into frequency domain:

\[ D(\omega, s) = \int e^{2ik|R_s,x|} A(\omega, s, x) d\omega V(x) dx \]

Choose origin of coordinates in antenna footprint, use far-field approximation

\[ |\gamma(s)| >> |x| \quad \Rightarrow \quad R_{s,x} = |\gamma(s) - x| \approx |\gamma(s)| - \hat{\gamma}(s) \cdot x + \cdots \]

\[ D(\omega, s) \approx e^{2ik|\gamma(s)|} \int e^{2ik\hat{\gamma}(s) \cdot x} A(\omega, s, x) V(x) dx \]

approximate by (function of \(\omega, s\)) (function of \(x\))

same as ISAR! use PFA
Each view gives slices of the ground reflectivity function in the time domain:

Reconstruct a function from its integrals over lines

use FBP
Outline

9. antenna theory
10. spotlight SAR
11. stripmap SAR
   (a) construction of imaging operator
   (b) a shortcoming of the continuum model
   (c) resolution
   (d) analysis of singularities
12. understanding SAR images
13. the state of the art
   (a) SAR interferometry
14. current research and open problems
Stripmap SAR

- Flight track
- Antenna
- $v$ (Antenna velocity)
- $\theta_{nr}$ (Near-range incidence)
- $x$ (Along-track direction)
- $R_s$ (Slant-range swath)
- $y$ (Across-track direction)
Or
data is of the form

\[ d(t, s) = \int \int \ e^{-i\omega(t-2|R_s,x|/c)} A(\omega, s, x) d\omega V(x) d\mathbf{x} =: F[V](t, s) \]

Cannot use far-field expansion as before
From \( d \), want to reconstruct \( V \).

- \( d \) is an oscillatory integral, to which techniques of microlocal analysis apply (\( F \) is a Fourier Integral Operator)
- similar to seismic inversion problem (with constant background but more limited data)
- \( d(t, s) \) depends on two variables.
  Assume \( V(x) = \tilde{V}(x_1, x_2) \delta(x_3 - h(x_1, x_2)) \).
    ground reflectivity function
- if \( A(\omega, s, x) = 1 \), want to reconstruct \( V \) from its integrals over spheres or circles (integral geometry problem)
Strategy for inversion scheme

G. Beylkin (JMP ’85)

Construct approximate inverse to $F$

Want $B$ (relative parametrix) so that $BF = I + (\text{smoother terms})$

Then $\text{image} = Bd \approx BF[V] = V + (\text{smooth error})$.

microlocal analysis $\Rightarrow$

a) method for constructing relative parametrix
b) theory $\Rightarrow BF$ preserves singularities

“local” $\longleftrightarrow$ location of singularities

“micro” $\longleftrightarrow$ orientation of singularities

singularities $\longleftrightarrow$ high frequencies

basic tool is method of stationary phase
Construction of imaging operator

recall
\[ d(s, t) = \int \int e^{-i\omega(t - 2|\mathbf{R}_s, \mathbf{x}|/c)} A(\omega, s, \mathbf{x}) d\omega V(z) dz \]

image = Bd where

\[ Bd(z) = \int \int e^{i\omega(t - 2|\mathbf{R}_s, z|/c)} Q(z, s, \omega) d\omega d(s, t) ds dt \]

where Q is to be determined.

- B has phase of \( F^* \) (\( L^2 \) adjoint)
- Compare:
  - inverse Fourier transform
  - inverse Radon transform
- This approach often results in exact inversion formula
Analysis of approximate inverse of $F$

$$I(z) = \int e^{i\omega(t-2|R_s,z|/c)}Q(z, s, \omega) d\omega \, d(s, t) dsdt$$

where $Q$ is to be determined below.

- Plug in expression for the data and do the $t$ integration:

$$I(z) = \int \int e^{i2k(|R_s,z|-|R_s,x|)}QA(\ldots) \, d\omega ds \, V(x) d^2x$$

point spread function

- Want $K$ to look like a delta function

$$\delta(z - x) = \int e^{i(z-x) \cdot \xi} d\xi$$

- Analyze $K$ by the method of stationary phase
The Stationary Phase Theorem

Assume:

\( a \) is a smooth function of compact support
\( \phi \) has only non-degenerate critical points (where \( \nabla \phi = 0, D^2 \phi \neq 0 \))

Then as \( \omega \to \infty \),

\[
\int e^{i\omega \phi(x)} a(x) d^n x =
\]

\[
\sum_{\{x^0: \nabla \phi(x^0) = 0\}} \left( \frac{2\pi}{\omega} \right)^{n/2} a(x^0) e^{i\omega \phi(x^0) + i(\pi/4) \text{sgn} D^2 \phi(x^0)} \frac{e^{i(\pi/4)\text{sgn} D^2 \phi(x^0)}}{\sqrt{\det D^2 \phi(x^0)}}
\]

\[+ O(\omega^{-n/2-1})\]

\( D^2 \phi \) = Hessian
\( \text{sgn} = \text{signature} = (\text{number of positive eigenvalues}) – (\text{number of negative eigenvalues}) \).
\[ K(z, x) = \int e^{i2k(|R_{s,z}| - |R_{s,x}|)} QA(\ldots) d\omega ds \]

main contribution comes from critical points

\[ |R_{s,z}| = |R_{s,x}| \]
\[ \hat{R}_{s,z} \cdot \dot{\gamma}(s) = \hat{R}_{s,x} \cdot \dot{\gamma}(s) \]

If \( K \) is to look like
\[ \delta(z - x) = \int e^{i(z-x) \cdot \xi} d^2 \xi, \]
we want critical points only when \( z = x \).

Antenna beam
should illuminate only one of the critical points ⇒ use side-looking antenna
\[ K(z, x) = \int e^{i2k(|R_{s,z}| - |R_{s,x}|)}QA(\ldots)d\omega ds \]

At critical point \( z = x \):

1. Do Taylor expansion of exponent about \( z = x \):

\[ 2k(|R_{s,z}| - |R_{s,x}|) = (z - x) \cdot \Xi(x, z, s, \omega) \]

near \( z = x \), \( \Xi(x, z, s, \omega) \approx 2k[\widehat{R}_{s,z}]T \)

2. Make (Stolt) change of variables

\[ (s, \omega) \rightarrow \xi = \Xi(x, z, s, \omega) \]

Then

\[ K(z, x) = \int e^{i(z-x) \cdot \xi}QA(\ldots) \left| \frac{\partial (s, \omega)}{\partial \xi} \right| d^2 \xi \]

Take \( Q = 1/(A |\partial (s, \omega)/\partial \xi|) \).

\[ \uparrow \]

Beylkin determinant.
Summary for single critical point at $z = x$

Form image by:

$$I(z) = \int e^{i\omega(t-2|R_{s,z}|/c)}Q(z, s, \omega)d\omega \, d(s, t)dsdt$$

where $Q = |\partial \xi/\partial (s, \omega)|/A, \quad \xi = 2k[\hat{R}_{s,z}]_T$

⇒ point spread function is

$$K(z, x) = \int_{\text{data manifold}} e^{i(z-x)\cdot \xi} d^2 \xi$$

Implications

1. singularities appear in correct location, with correct orientation
2. resolution determined by data manifold:
   - bandwidth, antenna pattern, and flight path
A shortcoming of this approach

continuum model avoids issues of range ambiguities and Doppler ambiguities

Figure 7.7 The ambiguity function of five coherent pulses ($T_p = 1$, $t_r = 0.2$): (a) 3-D view. (b) Contour plot.
source waveform SHOULD be a sequence of pulses

\[ j(t, y) = \sum_n j_n(t - T_n, y) = \sum_n \int e^{-i\omega(t-T_n)} J_n(\omega, y) d\omega \]

The transmitted (incident) field should be

\[ E^{in}(t, x, y) \approx \sum_n \int \frac{e^{-i\omega(t-T_n-|x-y|/c_0)}}{4\pi|x-y|} \hat{J}_n(\omega, x, y) d\omega \]

⇒ the scattered field is a sum over \( n \) ⇒ range ambiguities
reconstruction has \( \sum_n \) rather than \( \int \cdots ds \Rightarrow \) Doppler ambiguities

satellite systems use the antenna shape to avoid illuminating the ambiguous points

**Research question:** Can we avoid range and Doppler ambiguities by using mutually orthogonal coding of successive pulses?

all shifts must be mutually orthogonal
Outline

9. antenna theory
10. spotlight SAR
11. stripmap SAR
   (a) construction of imaging operator
   (b) a shortcoming of the continuum model
   (c) resolution
   (d) analysis of singularities
12. understanding SAR images
13. the state of the art
   (a) SAR interferometry
14. current research and open problems
Resolution

Resolution is determined by region in $\xi$-space where we have data.

Recall: $\xi \approx 2k[\hat{R}_{s,z}]_T$

Resolution via Fourier transforms

$$\int_{-b}^{b} e^{i\rho r} d\rho = \frac{2 \sin br}{r}$$

corresponds to resolution $2\pi/b$. 
Along-track resolution

\[ K(z, x) = \int e^{i(z-x) \cdot \xi} d\xi \text{ with } \xi \approx 2k[\hat{R}_{s,z}]_T. \]

flight track \( \gamma(s) = (0, s, H) \)

\( z - x = (0, z_2 - x_2, 0) \),

need only \( \xi_2 \)-range

\[ \xi_2 \approx 2k \frac{x_2 - \gamma(s)_2}{R} = k \frac{R}{R} 2(x_2 - s) \]

\( R = |x - \gamma(s)| \)

But

\[ 2 \max |x_2 - s| = \text{footprint width} \]

\[ = \frac{4\pi}{kL} R = \frac{2\lambda}{L} R \]

= effective length of the synthetic aperture

So \( \max |\xi_2| \approx \frac{k}{R} \frac{4\pi R}{k L} = \frac{4\pi}{L} \)

So resolution in along-track direction is \( \frac{2\pi}{4\pi/L} = \frac{L}{2} \)
Along-track resolution is $L/2$.

This is ...

- independent of range!
- independent of $\lambda$!
- better for small antennas!

When a point $z$ stays in the beam longer, the effective aperture for that point is larger.

In range direction, want broad frequency band ⇒ get largest coverage in $\xi$. 
Outline

9. antenna theory

10. spotlight SAR

11. stripmap SAR
   (a) construction of imaging operator
   (b) a shortcoming of the continuum model
   (c) resolution
   (d) analysis of singularities

12. understanding SAR images

13. the state of the art
   (a) SAR interferometry

14. current research and open problems
To understand why singularities appear in correct location, with correct orientation in the image, we need a little microlocal analysis.
“Microlocal” study of singularities

Singularities in reflectivity $\longleftrightarrow$ boundaries between different materials

Singularities have both a location and a direction

$(x^0, \xi)$ is not in the wavefront set of $f$ if for some smooth cutoff function $\phi$, the Fourier transform

$$\int f(x) \phi(x) e^{i\xi \cdot x} dx$$

decays rapidly in a neighborhood of the direction $\xi$.

To determine whether $(x^0, \xi)$ is in the wavefront set:

1. localize around $x^0$
2. Fourier transform
3. look at decay in direction $\xi$
Example: a small “point” scatterer $V(x) = \delta(x)$

localize around any $x \neq 0 \Rightarrow \phi V \equiv 0$ (decays rapidly)
$\Rightarrow (x, \xi) \notin WF(\delta)$

localize around $x = 0 \Rightarrow \phi V = \delta \propto \int e^{i x \cdot \xi} 1 d\xi$

i.e., $\mathcal{F}V = 1$ (does not decay in any direction)

$WF(\delta) = \{(0, \xi) : \text{all } \xi \neq 0\}$. (has singularities in all directions)
Example: a wall: \( V(x) = \delta(x \cdot \hat{\nu}) \propto \int e^{i x \cdot \hat{\nu} \rho} d\rho \)

localize around any point \( x \) with \( x \cdot \hat{\nu} \neq 0 \) ⇒ \( \phi V \) is smooth ⇒ Fourier transform decays rapidly ⇒ \( (x, \xi) \notin WF(V) \)

⇒ \( FV(\xi) = \int \delta(\xi - \hat{\nu} \rho) d\rho = \begin{cases} 1 & \xi \propto \hat{\nu} \\ 0 & \text{otherwise} \end{cases} \)

singularity in the direction \( \hat{\nu} \)
The Fourier transform does not decay rapidly in direction \( \hat{\nu} \).
\[ I(z) \sim \int K(z, x) V(x) \, dx \]

where

\[ K(z, x) = \int_{\text{data manifold}} e^{i(z-x) \cdot \xi} \, d\xi \]

is the kernel of a \textbf{pseudodifferential operator}.

Pseudodifferential operators have the \textbf{pseudolocal} property:

\[ WF(Ku) \subseteq WF(u) \]

- put singularities in the correct location
- do not change orientation of singularities
- singularities CAN disappear
The pseudolocal property

Example: \( V(x) = \delta(x \cdot \hat{\nu}) \propto \int e^{ix \cdot \hat{\nu} \rho} d\rho \)

Then \( \int K(z, x) \delta(x \cdot \hat{\nu}) dx \propto \int K(z, x) e^{ix \cdot \rho \hat{\nu}} d\rho dx \)

\[
= \int e^{i(z-x) \cdot \xi} \chi(z, x, \xi) e^{ix \cdot \rho \hat{\nu}} d\xi dx d\rho,
\]

change variables \( \xi \rightarrow \rho \tilde{\xi} \)

large-\( \rho \) stationary phase reduction in \( x \) and \( \tilde{\xi} \)

\( \phi = \rho[(z - x) \cdot \tilde{\xi} + x \cdot \hat{\nu}] \)

leading order contribution comes from:

\[
d\phi/d\tilde{\xi} \Rightarrow x = z, \quad d\phi/dx = 0 \Rightarrow \tilde{\xi} = \hat{\nu}
\]

(correct location) \hspace{1cm} (correct orientation)

\[
\int K(z, x) \delta(x \cdot \hat{\nu}) dx \propto \int \chi(z, z, \rho \hat{\nu}) e^{iz \cdot \rho \hat{\nu}} d\rho + \text{(smoother)}
\]

\( \Rightarrow \) singularities are microlocally correct
What about the extraneous critical points?

change of variables $(s, \omega) \rightarrow \xi$ can NOT be done.

stationary phase analysis at extraneous critical point \(\Rightarrow\)

\[
K(z, x) \approx \frac{e^{i\pi/4}}{(2\pi)^{3/2}} \int_{\text{survey}} \frac{1}{|\omega|^{1/2}} \frac{A(x, s, 2|R_{s,x}|/c_0, \omega)}{A(z, s, 2|R_{s,z}|/c_0, \omega)} 
\cdot \frac{1}{|\phi''|^{1/2}(z, x, s)} \left| \frac{\partial \xi}{\partial (s, \omega)} \right| (z, z, s, \omega) d\omega
\]

where \(s = s(z, x)\) satisfies criticality conditions

where \(\phi'' \propto \text{curvature of flight track}\)

compare with

\[
K(x, x) \approx \frac{1}{(2\pi)^2} \int_{\text{survey}} \left| \frac{\partial \xi}{\partial (s, \omega)} \right| (x, x, s, \omega) d\omega ds
\]

\(\Rightarrow\) increasing curvature decreases artifact strength
9. antenna theory
10. spotlight SAR
11. stripmap SAR
   (a) construction of imaging operator
   (b) a shortcoming of the continuum model
   (c) resolution
   (d) analysis of singularities
12. understanding SAR images
13. the state of the art
   (a) SAR interferometry
14. current research and open problems
Figure 7.13 Principles of imaging: (a) optical image of the Washington Monument; (b) SAR image of the monument.
Outline

9. antenna theory
10. spotlight SAR
11. stripmap SAR
   (a) construction of imaging operator
   (b) a shortcoming of the continuum model
   (c) resolution
   (d) analysis of singularities
12. understanding SAR images
13. the state of the art
   (a) SAR interferometry
14. current research and open problems
State of the Art

- motion compensation (autofocus algorithms)
- moving target indicator radar
- polarimetric SAR
• SAR interferometry

Fig. 3.3a. Compare this observed coseismic interferogram for the Landers earthquake [Massonnet et al., 1993] with the synthetic interferogram in Fig. 3.3b. One cycle of color represents 28 mm of change in range. Black segments depict the fault geometry as mapped in the field. Both this image and Fig. 3.3b cover a 90-by-110-km area from April 24 to August 7, 1992.
Second pass: measures phase ($\phi_1$) for each pixel for time ($t_1$).

First pass: measures reference phase ($\phi_0$) for each pixel for time ($t_0$).

Phase ($\phi$): each color represents the phase ($\phi$) of a wave.

Displacement: pixel A moves about 1.5 wavelengths (~4.2 cm) toward the satellite, with respect to pixel B.

InSAR image: phase difference ($\phi_1 - \phi_0$) for each pixel during time interval ($t_1 - t_0$).

Random text for filler.
SAR interferometry

Take \( j(t, y) = p(t)\tilde{j}(y) \) with \( p(t) = \tilde{p}(t)e^{-i\omega ct} \)

\[
p_{\text{rec}}(t, s) = \int \int e^{-i\omega(t-2|R_{s,x}|/c)} P(\omega) \tilde{A}(s, x) d\omega V(x) dx
\]

\[
= \int \tilde{p}(t-2|R_{s,x}|/c)e^{-i\omega c(t-2|R_{s,x}|/c)} \tilde{A}(s, x)V(x) dx
\]

use \( V(x) = \tilde{V}(x_1, x_2)\delta(x_3 - h(x_1, x_2)) \)

Write \( x = x_T + h\hat{e}_3 \) with \( x_T = (x_1, x_2, 0) \); use far-field expansion

\[
|R_{s,x}| = |x-\gamma(s)| = |x_T + h\hat{e}_3 - \gamma(s)| = |x_T - \gamma(s)| + h\hat{e}_3 \cdot (x_T - \gamma(s)) + \cdots
d(x_T)
\]

\[
\Rightarrow p_{\text{rec}}(t, s) = \int \tilde{p}(t-2|R_{s,x_T}|/c)e^{-i\omega c(t-2|R_{s,x_T}|/c)} \tilde{A}(s, x) \left[ e^{-2ik_c d(x_T)} \tilde{V}(x_T) \right] d^2 x_T
\]

This is signal that would have been obtained from a flat earth with reflectivity function \( \left[ e^{-2ik_c d(x_T)} \tilde{V}(x_T) \right] \)
1. form two complex images of \( e^{-2ikc d(z_T)} \tilde{V}(z_T) \)
   in \( j \)th image, pixel at \( z_T \) is at range \( r_j = |\gamma_j - z_T| \)
   the true range is \( R_j = r_j - d_j \approx |\gamma_j - z| \)
1. form two complex images of \( e^{-2ikc d(z_T)} \tilde{V}(z_T) \) in \( j \)th image, pixel at \( z_T \) is at range \( r_j = |\gamma_j - z_T| \) the true range is \( R_j = r_j - d_j \approx |\gamma_j - z| \)

2. co-register the images so that \( r_1 = r_2 \)
   \[ \Rightarrow R_2 = R_1 + (d_1 - d_2) \]

3. multiply the complex conjugate of one image by the other \( \rightarrow \) complex image whose phase at \( z_T \) is \( 2k_c [d_1(z_T) - d_2(z_T)] \).

4. solve phase unwrapping problem to get \( d_1(z_T) - d_2(z_T) \)

5. use the law of cosines to get the angle of elevation \( \theta \):
   \[ R_2^2 = R_1^2 + b^2 - 2bR_1 \cos(\beta + \pi/2 - \theta) \]

6. height = \( H - R_1 \cos \theta \)
Outline

9. antenna theory
10. spotlight SAR
11. stripmap SAR
   (a) construction of imaging operator
   (b) a shortcoming of the continuum model
   (c) resolution
   (d) analysis of singularities
12. understanding SAR images
13. the state of the art
   (a) SAR interferometry
14. current research and open problems
Current research and open problems

• Waves that propagate through complex media
  – foliage-penetrating SAR
    * want bare earth topography
    * find objects under trees
    * estimate forest biomass, trunk volume, tree health
    * must (?) use low frequencies ⇒
      • resolution is limited; how can we identify objects from low-resolution images?
      • antenna directivity is poor ⇒ left-right ambiguities
    * vehicles parked near tree trunks ⇒ multiple scattering
  – imaging through the ionosphere
  – ground-penetrating radar (GPR or Gpen)
    * ice
    * land mines, unexploded ordinance (UXO)
Are there vehicles parked under the trees?
* utility pipes, bridge decks
* earth is in near-field of antenna ⇒ antenna properties change depending on what is in the earth
  - imaging in urban areas
    multipath scattering, interference
  - through-the-wall imaging
    hostage situations, urban warfare
  - polarimetric scattering in random media
  - dispersive media

• exploiting emerging radar capabilities
  - agile antennas
    waveform design and scheduling
  - multistatic scenarios
  - sparse, random networks of antennas
  - swarms of UAVs
– unattended ground sensors
– sources of opportunity
– staring radar
– simultaneous tracking and imaging of moving targets
– 3D imaging
Are there cars driving on these streets?

Where are they going?

Can a computer track them?
Locate and image the moving objects
M-47 Tanks On Kirtland AFB
Comparison of Resolutions At Actual and 4x Enlarged Views

Resolution = 1 Meter
Resolution = 1 Foot
Resolution = 4 Inches

GENERAL ATOMICS
LYNX SAR
• Image interpretation
  – Thickness of ice, species of trees, surface roughness, …
  – Automatic Target Recognition (ATR) school bus or tank? identify airplanes
  – Sensor fusion, use of multiple frequency bands
  – Identification of singularities in bandlimited data or image

• Theoretical issues
  – Reconstruction for full nonlinear inverse problem (avoid the Born approximation!)
    * limited aperture
    * time domain (including dispersion!)

• Nonlinear calculus for singularities?
• Uniqueness theorem for backscattering
What information can we get from the image?

image of same scene at two different frequencies
Where should a ship sail to get through the ice?
Radar imaging is a field that is ripe for mathematical attention!