

INTRODUCTION TO DIFFERENTIAL EQUATIONS, TEST 2
Sections 2 and 4, Spring 1999

Section _____ Name _____

Instructions. You are allowed to use one $8\frac{1}{2} \times 11$ inch sheet of paper of notes. No calculators, computers, books, or cellular phones are allowed. Do not collaborate in any way. In order to receive credit, your answers must be clear, legible, and coherent.

1. (20 points) The functions $u_n(x, y) = \sin(nx) \sinh(\sqrt{n^2 - 1}y)$, $n = 1, 2, \dots$ satisfy

$$\partial_{xx}u + \partial_{yy}u + u = 0 \tag{1}$$

in the set $\{(x, y) : 0 < x < \pi, 0 < y < 2\}$, together with the boundary conditions

$$u(0, y) = 0 = u(\pi, y) \quad \text{for } 0 < y < 2 \tag{2}$$

$$u(x, 0) = 0 \quad \text{for } 0 < x < \pi. \tag{3}$$

Find a function (perhaps in the form of an infinite series) satisfying (1), (2), and (3) together with the additional condition $u(x, 2) = 3$ for $0 < x < \pi$. (Hint: In other words, you are given the solution to Step A; now carry out Step B.)

$u(x, y) =$ _____

2. (60 points) A vibrating string of length π that is CLAMPED at one end and FREE at the other end satisfies the following conditions. Find the solution u .

$$u_{xx} = u_{tt}$$

$$u(0, t) = 0 = u_x(\pi, t) \quad \text{for } t > 0$$

$$u(x, 0) = 5 \sin(3x/2) \quad \text{for } 0 < x < \pi$$

$$u_t(x, 0) = 0 \quad \text{for } 0 < x < \pi$$

Hint: You will not get the usual Fourier series, but you can still solve the problem with these special initial conditions.

$u(x, t) =$ _____

3. (20 points) Sketch the function $f(x)$ to which the Fourier series converges. Pay special attention to points of discontinuity.

$$f(x) = \sum_{n=1}^{\infty} b_n \sin 2n\pi x$$

where

$$b_n = 8 \int_0^{1/4} x \sin 2n\pi x dx$$