Math 4600

Homework 1  Due Feb. 1

Write your solutions in separate paper and clearly mark the problem number.

- Do the following problems from the text.
  33, 34 in Section 1.7.
  1 in Section 2.1.
  1, 2, 3, 9, 12 in Section 2.2.
  3, 6, 8, 17, 20, 25 in Section 2.3.
  9, 14 in Section 2.4.

- In this problem we review some linear algebra. The questions are in bold face.

We use a basis to denote the coordinates of points or vectors. Suppose we are using the standard basis \( S = \{ \vec{e}_1, \vec{e}_2, \vec{e}_3 \} \) where \( \vec{e}_1 = (1, 0, 0), \vec{e}_2 = (0, 1, 0), \vec{e}_3 = (0, 0, 1) \). Now consider a vector \( \vec{v} = (-4, 2, 3) \). Note that we can span \( \vec{v} \) by \( S \) as

\[
\vec{v} = (-4)\vec{e}_1 + 2\vec{e}_2 + 3\vec{e}_3
\]

That is, \((-4, 2, 3)\) is the coefficients in the span of \( \vec{v} \) by \( S \). To be precise \((-4, 2, 3)\) is the coordinate of \( \vec{v} \) with respect to \( S \) and we can write \( \vec{v}_S = (-4, 2, 3) \). If we choose a different basis the coordinate of \( \vec{v} \) will change also.

(L-1) Let \( \mathcal{B} = \{ \vec{w}_1 = (1, 1, 1), \vec{w}_2 = (2, 0, 1), \vec{w}_3 = (-1, 0, 0) \} \) be another basis and find \( \vec{v}_B \).

To find a new coordinate in the previous question we needed to solve a system of linear equations. The change of basis would have been simpler if \( \mathcal{B} \) was an orthonormal basis. In general if \( \mathcal{U} = \{ \vec{u}_1, \vec{u}_2, \ldots, \vec{u}_n \} \) is an orthonormal basis then we can span a vector \( \vec{v} \) as

\[
\vec{v} = (\vec{v} \cdot \vec{u}_1)\vec{u}_1 + (\vec{v} \cdot \vec{u}_2)\vec{u}_2 + \ldots + (\vec{v} \cdot \vec{u}_n)\vec{u}_n
\]

Therefore we often orthonormalize a given basis.

(L-2) Orthonormalize \( \mathcal{B} \) using the Gram-Schmidt process.