1. Given \(|f(x) - f(y)| \leq M |x-y|^{a}, \quad a > 1\). Prove that \(f = \text{const}\).

Since
\[
\left| \frac{f(x+h) - f(x)}{h} \right| \leq \frac{M |h|^a}{|h|} = M |h|^{a-1}
\]

and \(\lim_{h \to 0} M |h|^{a-1} = 0\), then \(f'(x) = 0\) for any \(x\).

Thus \(f(x) - f(x_0) = f'(c)(x-x_0) = 0\), i.e. \(f(x) \equiv f(x_0)\)

2. Prove that \(x^k\) is continuously differentiable.

For \(x > 0\), \(x^k\) is cont. differentiable, for \(x < 0\), \(x^k = 0\) is cont. differentiable.

Consider \(x_0 = 0\), for \(h > 0\), \(\frac{h^k}{h} = h^{k-1}\), \(\lim_{h \to 0^+} h^{k-1} = 0\); for \(h < 0\), \(\frac{h^k}{h} = 0\), \(\lim_{h \to 0^-} 0 = 0\).

Thus \(\frac{h^k}{h} = \begin{cases} kx^{k-1} & x > 0 \\ 0 & x \leq 0 \end{cases}\)

Therefore \((x^k)' = 0\) at \(x = 0\).

This function is continuous.
3. Taylor expansion of \( f(x) = (x^2+1)^{25} \) to order 3.

1) About \( x_0 = 0 \) \( x^2+1 \) is its own Taylor expansion

\[
(x^2+1)^{25} = 1 + 25x^2 + \frac{25 \cdot 2y}{1 \cdot 2} x^4 + \ldots
\]

Truncate to retain 3 and lower terms

\[
(x^2+1)^{25} = 1 + 25x^2 + o(x^3)
\]

2) About \( x_0 = 1 \). Just use the formula

\[
f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)(x-x_0)^2}{2!} + \ldots
\]

to get:

\[
f(x) = 2^{25} \left[ 1 + 25(x-1) + \frac{25 \cdot 13}{2} (x-1)^2 + \frac{25 \cdot 26 \cdot 12}{3!} (x-1)^3 \right] + o(x-1)^3.
\]

(Hint: Ignore the hint I gave)

4. Computation of \( (x^{1/k})' \).

\[
x^{1/k} = y \Rightarrow x = y^k.
\]

\[
\frac{dy}{dx} = \frac{1}{k y^{k-1}} = \frac{1}{k} y^{1-k} = \frac{1}{k} x^{\frac{1}{k}(1-1)}
\]

\[
= \frac{1}{k} x^{\frac{1}{k}-1}
\]