April 23

8.4 Annuities, Stocks and Bonds ---- Systematic Savings

Annuity = sequence of equal payments made at equal time periods
        i.e. depositing $1000 at the end of every year into an IRA

Value of an annuity = the sum of all the deposits plus all interest paid.

\[
A = P \left[ \frac{(1 + r/n)^{nt} - 1}{(r/n)} \right]
\]

Where \( A \) = future value, \( P \) = deposit made at the end of each compounding period,
\( n \) = # of times compounded per year, \( t \) = # of years, \( r \) = annual interest rate
At age 25, to save for retirement, you decide to deposit $75 at the end of each month in an IRA that pays 6.5% interest compounded monthly.

a) How much $ will be in the IRA at age 65? 40 years later

\[ A = \frac{75 \left[ \left( 1 + \frac{.065}{12} \right)^{12 \times 40} - 1 \right]}{\left( \frac{.065}{12} \right)} \]

\[ = 75 \left[ 12.3696 \ldots \right] = \frac{971,271.42}{.005416} \]

b) How much of the $ in the IRA is interest?

Total amount deposited = \((75)(12)(40) = 36,000\)

\[ \text{Interest} = 171,271 - 36,000 = 135,271 \]
You would like $4000 in 4 years for a special vacation. You make deposits every 6 months in an annuity that pays 7% compounding semi-annually.

a) How much (to the nearest $) should you deposit at the end of every 6 months to meet your goal?

\[
P = \frac{4000 \left( \frac{0.07}{2} \right)}{\left( 1 + \left( \frac{0.07}{2} \right) \right)^{2(4)} - 1}
\]

\[
P = \frac{4000 \times 0.035}{0.316809...} = 441.906 \approx 442
\]

b) How much of the $4000 is deposits and how much is interest?

Total deposit amount = (442)(2)(4) = $3536

Interest = \[A - \text{total deposit}\]

\[= 4000 - 3536 = 464\]
Investments

Cash investments: bank, savings, CD

STOCKS - shares of ownership in a company

two ways to make $:

1. sell shares for more
   $ than you bought them < $ CAPITIAL GAIN (LOSS)

2. DIVIDENDS = company distribute profits to shareholders

BONDS - commitment by company to pay face value + interest on a "loan" to company
Reading Stock Tables

Daily newspapers and online services give current stock prices and other information about stocks.

- **52-week high** refers to the highest price at which a company traded during the past 52 weeks.
- **52-week low** refers to the lowest price at which a company traded during the past 52 weeks.
- **Stock** refers to the company name.
- **SYM** refers to the symbol the company uses for trading.
- **Div** refers to dividends paid per share to stockholders last year.
- **Yld%** stands for percent yield.
- **Vol100s** stands for sales volume in hundreds.
Reading Stock Tables

• **Hi** stands for the highest price at which the company’s stock traded yesterday.

• **Low** stands for the lowest price at which the company’s stock traded yesterday.

• **Close** stands for the price at which shares traded when the stock exchanged closed yesterday.

• **Net Chg** stands for net change.

• **PE** stands for the price-to-earnings ratio.

\[
PE \text{ ratio} = \frac{\text{Yesterday's closing price per share}}{\text{Annual earnings per share}}
\]
<table>
<thead>
<tr>
<th>52-week High</th>
<th>Low</th>
<th>Stock</th>
<th>SYM</th>
<th>Div</th>
<th>Yld %</th>
<th>PE</th>
<th>Vol 100s</th>
<th>Hi</th>
<th>Lo</th>
<th>Close</th>
<th>Net Chg</th>
</tr>
</thead>
<tbody>
<tr>
<td>42.38</td>
<td>22.50</td>
<td>Disney</td>
<td>DIS</td>
<td>.21</td>
<td>.6</td>
<td>43</td>
<td>115900</td>
<td>32.50</td>
<td>31.25</td>
<td>32.50</td>
<td>...</td>
</tr>
</tbody>
</table>

Dividend = \( \frac{.21}{100} \) or \( \frac{21}{1000} \) /share

If owned 1000 shares = get $210

\( (.21 \times 1000) \)

Yesterday closed at same price as the day before
Annual earnings per share = \(\frac{\text{Yesterday's close}}{\text{PE ratio}}\)

= \(\frac{\$32.50}{43}\)

= \$0.76
Fixed Installment Loans -- (car, school, house)

A loan that has a schedule of paying a fixed amount each period for the entire term of the loan.

You need to be able to determine the Amount financed (cash price – down payment), the total installment price (monthly payments + down payment), and the finance charge (the total installment price – cash price).

A new car costs $14000. You finance the car by paying $280 down and $315 per month for 60 months.

a) What is the amount financed?

$14000 - $280 = $13720

b) What is the total installment price?

$60(315) + $280 = $19,180

c) What is the finance charge?

$19,180 - $14000 = $5,180
So what interest rate did you pay?

1) Compute the finance charge per $100

\[
\text{Finance charge} \times 100 = \frac{5180}{13720}. 100 = \$37.76
\]

2) Look in Table: find row for # payments and find dollar amount closest to answer above.

**TABLE 8.3 ANNUAL PERCENTAGE RATE (APR) FOR MONTHLY PAYMENT LOANS**

<table>
<thead>
<tr>
<th>Number of Monthly Payments</th>
<th>10.0%</th>
<th>10.5%</th>
<th>11.0%</th>
<th>11.5%</th>
<th>12.0%</th>
<th>12.5%</th>
<th>13.0%</th>
<th>13.5%</th>
<th>14.0%</th>
<th>14.5%</th>
<th>15.0%</th>
<th>15.5%</th>
<th>16.0%</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Finance charge per $100 of amount financed)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>$2.94</td>
<td>$3.08</td>
<td>$3.23</td>
<td>$3.38</td>
<td>$3.53</td>
<td>$3.68</td>
<td>$3.83</td>
<td>$3.97</td>
<td>$4.12</td>
<td>$4.27</td>
<td>$4.42</td>
<td>$4.57</td>
<td>$4.72</td>
</tr>
<tr>
<td>12</td>
<td>5.50</td>
<td>5.78</td>
<td>6.06</td>
<td>6.34</td>
<td>6.62</td>
<td>6.90</td>
<td>7.18</td>
<td>7.46</td>
<td>7.74</td>
<td>8.03</td>
<td>8.31</td>
<td>8.59</td>
<td>8.88</td>
</tr>
<tr>
<td>48</td>
<td>21.74</td>
<td>22.90</td>
<td>24.06</td>
<td>25.23</td>
<td>26.40</td>
<td>27.58</td>
<td>28.77</td>
<td>29.97</td>
<td>31.17</td>
<td>32.37</td>
<td>33.59</td>
<td>34.81</td>
<td>36.03</td>
</tr>
<tr>
<td>60</td>
<td>27.48</td>
<td>28.96</td>
<td>30.45</td>
<td>31.96</td>
<td>33.47</td>
<td>34.99</td>
<td>36.52</td>
<td>38.06</td>
<td>39.61</td>
<td>41.17</td>
<td>42.74</td>
<td>44.32</td>
<td>45.91</td>
</tr>
</tbody>
</table>
Some fixed loans can be paid off early with no penalty ... this reduces the interest you pay (unearned interest)

**METHODS FOR COMPUTING UNEARNED INTEREST**

Unearned interest is the amount by which a loan’s finance charge is reduced when the loan is paid off early.

**Actuarial Method**

\[
\begin{align*}
    u &= \frac{kRV}{100 + V} \\
    u &= \text{unearned interest} \\
    k &= \text{remaining number of scheduled payments (excluding current payment)} \\
    R &= \text{regular monthly payment} \\
    V &= \text{finance charge per$100 (from the APR table) for a loan with the same APR and } k \text{ monthly payments}
\end{align*}
\]

**Rule of 78**

\[
\begin{align*}
    u &= \frac{k(k + 1)}{n(n + 1)} \times F \\
    u &= \text{unearned interest} \\
    k &= \text{remaining number of scheduled payments (excluding current payment)} \\
    n &= \text{original number of payments} \\
    F &= \text{original finance charge}
\end{align*}
\]
Back to our car example: $280 down, $315 per month for 60 months.

You decide to pay off car at the 24th payment.

a) Use the actuarial method to determine interest saved

\[
u = \frac{k R V}{100 + \sqrt{v}}
\]

\[
= \frac{36(315)(22.17)}{122.17}
\]

\[
= \$2057.85 \quad \text{interest saved}
\]

b) What is the payoff amount?

\[
\text{payoff amount} = (\text{payment #24}) + (\text{Total remaining payments after 24})
\]

\[
= \$315 + (36)(\$315) - \frac{\text{Interest Saved}}{2057.85}
\]

\[
= \$9567.15
\]
Back to our car example: $280 down, $315 per month for 60 months.

You decide to pay off car at the 24th payment.

a) Use the Rule of 78 to determine interest saved

\[
I = \frac{k(k+1)}{n(n+1)} \times F
\]

\[
= \frac{36(37)}{60(61)} \times 5180 \approx 1885.18
\]

b) What is the payoff amount?

\[
315 + 36(315) - 1885.18 = \boxed{9769.82}
\]
Open-end Installment Loans --- Credit Cards --- revolving credit

Interest is computed using the Simple Interest Formula

\[ I = Prt \]

where \( r \) is the **monthly** interest rate!

Three methods to calculate monthly interest:

For all three methods, \( I = Prt \), where \( r \) is the monthly rate and \( t \) is one month.

**Unpaid balance method:** The principal, \( P \), is the balance on the first day of the billing period less payments and credits.

**Previous balance method:** The principal, \( P \), is the unpaid balance on the first day of the billing period.

**Average daily balance method:** The principle, \( P \), is the *average daily balance*. This is determined by adding the unpaid balances for each day in the billing period and dividing by the number of days in the billing period.
A credit card has a monthly rate of 1.8% (the APR is $1.8 \times 12 = 21.6\%$). The January 1 – January 31 itemized billing is as follows:

- **January 1**: unpaid balance of $6800
- **January 8**: payment of $500 received

No purchases or cash advances were made.

Payment due date is February 9.

Unpaid balance: $I = Prt$  
$I = (6800 - 500) (0.018)(1) = 113.40$

Previous balance: $I = Prt$  
$I = (6800)(0.018)(1) = 122.40$

Avg daily balance: $P = \frac{6800 (7) + 6300 (24)}{31} = 6412.90$

$I = (6412.90)(0.018)(1) = 115.43$
Discuss Amortization – payments for fixed loans (car/home) divided between interest and principal and proportion to each changes during the life of the loan.

Chap 8.6 - specifically amortization of home loans