1. Determine whether the set of vectors

\[ \begin{align*}
\mathbf{x}^{(1)}(t) &= \begin{pmatrix} 2e^{2t} \\ 2e^{2t} \\ 0 \end{pmatrix}, \\
\mathbf{x}^{(2)}(t) &= \begin{pmatrix} e^{2t} \\ -2e^{2t} \\ e^{2t} \end{pmatrix}, \\
\mathbf{x}^{(3)}(t) &= \begin{pmatrix} e^{2t} \\ 3e^{2t} \\ -e^{2t} \end{pmatrix}
\end{align*} \]

is linearly dependent for \(-\infty < t < \infty\). If so, find the linear relation among them. Justify your conclusion.

2. Consider \( \mathbf{x}^{(2)}(t) = \begin{pmatrix} 2 \\ t \end{pmatrix}, \mathbf{x}^{(2)}(t) = \begin{pmatrix} t \end{pmatrix} \).

(a) Compute the Wronskian of \( \mathbf{x}^{(1)} \) and \( \mathbf{x}^{(2)} \).
(b) For what values \( t = t_0 \) are the constant vector \( \mathbf{x}^{(2)}(t_0) \) and \( \mathbf{x}^{(2)}(t_0) \) linearly independent?
(c) In what interval(s) are \( \mathbf{x}^{(1)}(t) \) and \( \mathbf{x}^{(2)}(t) \) linearly independent?
(d) What conclusions can you draw about the coefficients in the system of linear homogeneous DE's satisfied by \( \mathbf{x}^{(1)}(t) \) and \( \mathbf{x}^{(2)}(t) \)?
(e) Find the system of equations and verify the conclusions of part (d).

[SEE Prob. 6 & 7, p. 369, FOR SIMILAR PROBLEMS]

3. Consider the nonhomogeneous linear system

\[ \begin{align*}
\frac{d}{dt} \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 4 & 5 \\ -5 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -2 \\ 7 \end{pmatrix} \\
(\ast)
\end{align*} \]

(a) Find the unique critical point of \((\ast)\) and determine its type and stability.

[If the c.p. is \((x_0, y_0)\), make the substitution \( x = u + x_0, \ y = v + y_0 \) to transform \((\ast)\) into a linear homogeneous system. Then solve.]
(b) Sketch several trajectories of \((\ast)\) in the \(x, y\)-plane.
(c) Given the expressions for \(u(t)\) and \(v(t)\), find the general solution \(x(t), y(t)\) of system \((\ast)\). AN EQUIVALENT PROCEDURE FOR SOLVING \((\ast)\)
4. Consider the nonlinear autonomous system

\[ x' = x(3 - x - y), \quad y' = y(-2 - y + x) \quad (**) \]

(a) Find all equilibrium solutions (critical points).
(b) There is one c.p. for which \( x \) and \( y \) are both positive. For this c.p., find the linear system of differential equations that approximates (**) near this point.
(c) With respect to the linearized system found in (b), determine the type & stability of the c.p.
(d) With respect to the nonlinear system (**) what are the type and stability of the c.p.? Explain.
(e) Sketch a few trajectories near this c.p. in the \( x,y \)-plane.

\[ \text{DUE: Thursday, 4/20 by noon in Leslie's mailbox.} \]