1. $\min \sum_{i\in M}\sum_{j\in N} h_{ij} y_{ij} + \sum_{j\in N} c_j x_j$

$$\text{s.t.} \quad \sum_{j\in N} y_{ij} \le a_i \quad \text{for } i \in M \qquad (1)$$

$$\sum_{i\in M} y_{ij} \le b_j x_j \quad \text{for } j \in N \qquad (2)$$

$$y_{ij} \le \min\{a_i, b_j\} x_j \quad \text{for } i\in M, j \in N. \qquad (3)$$

$$y \in \mathbb{R}^{mn}, \quad x \in \mathbb{B}^n$$

(Note that h_{ij} may be negative. A more standard formulation would have $=$ in (1).)

(a) Relax (1): Then Lagrangian relaxation separates into different problems for each $j \in N$. Also, can solve the problem by solving LP relaxation. So only as strong as LP relaxation.
Same applies to relaxing (1) & (2), or relaxing (1) & (3); solving Lagrangian dual is harder with more constraints dualized.

(b) Relax (2): Then similar to uncapacitated facility location problem. Hard to solve Lagrangian relaxation, but should give a better bound than LP.

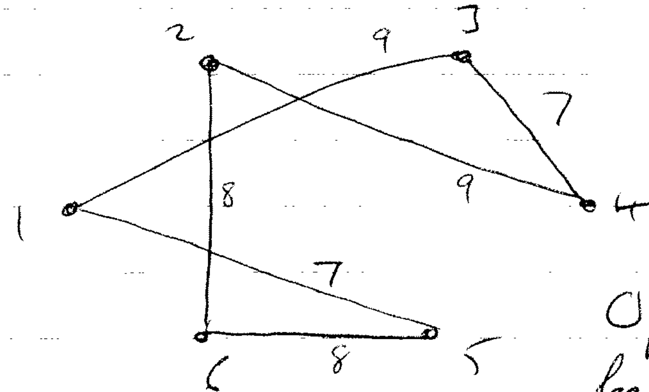
(c) Relax (3): (3) is redundant in IP, so this is equivalent to the IP.

(d) Relax (2) & (3): Get LP, so easy to solve relaxation, but no improvement on LP relaxation.

2. With $\lambda = (0,0,0,-1,2,-2)$, edge costs become:

	1	2	3	4	5	6
1	–	10	9	13	7	12
2	10	–	11	9	9	8
3	9	11	–	7	11	12
4	13	9	7	–	10	10
5	7	9	11	10	–	8
6	12	8	12	10	8	–

1-tree:



Optimal, since feasible in TSP.

Cost: $9+7+8+9+7+8=48$

Add penalty: $2\left(-\sum \lambda_i\right) = 2.$

Total cost $\boxed{50.}$

Other possible choices for λ: $(0,0,0,0,4,-1)$
$(0,0,0,-1,3,-2)$
...

3. Choose $\lambda = 0$:

One soln: A goes $c - a - b - j - l$.

B goes $h - g - p - l$.

Total value 7.

Feas in IP, and optimal for IP, and solves LP relaxation of problem.