TRUE/FALSE QUESTIONS
CARDINALITY, INDUCTION, AND DECIMAL REPRESENTATIONS
Chapters #5,6,7

(1) If the cardinality of a set, \( S \), is the positive integer, \( n \), then the cardinality of \( 2^S \) is equal to \( 2^n \).

(2) Let the cardinality of the finite sets, \( S_m \) and \( S_n \) be \( m \) and \( n \) respectively. If \( m < n \) there exists a function from \( S_m \) onto \( S_n \).

(3) Let the cardinality of the finite sets, \( S_m \) and \( S_n \) be \( m \) and \( n \) respectively. If \( m < n \) there exists a function from \( S_n \) onto \( S_m \).

(4) If there exists a function from a non-empty set, \( S \), onto a non-empty set, \( T \), then the cardinality of \( S \) is greater than the cardinality of \( T \).

(5) The cardinality of a non-empty set, \( S \), is always less than the cardinality of \( 2^S \).

(6) The cardinality of the positive integers is the same as the cardinality of all of the integers.

(7) The cardinality of \( \mathbb{R} \) is greater than the cardinality of the rational numbers.

(8) The cardinality of the set of all real-valued functions defined on \( \mathbb{R} \) is greater than the cardinality of \( \mathbb{R} \).

(9) A non-empty subset of a countable set is countable.

(10) Let \( A \) and \( B \) denote non-empty sets such that \( A \cap B = \emptyset \). Then the cardinality of \( A \cup B \) is greater than the cardinality of \( A \).

(11) Every subset of a finite set is finite.

(12) Every infinite set contains a countably infinite subset.

(13) A countable set of countable sets is countable.
(14) Let $A$ and $B$ be non-empty finite sets. Then $|A \cup B| = |A| + |B|$.

(15) Let $A$ and $B$ be non-empty finite sets. Then $|A \times B| = |A| \cdot |B|$.

(16) A finite set has exactly one cardinality.

(17) An infinite set contains finite subsets of cardinality $n$ for all $n \in \mathbb{Z}^+$.

(18) Let $A$ and $B$ denote non-empty sets such that $A$ is properly contained in $B$. Then the cardinality of $B$ is greater than the cardinality of $A$.

(19) The cardinality of the empty set is $0$.

(20) Let $A$ and $B$ denote non-empty finite sets such that $A$ is properly contained in $B$. Then the cardinality of $B$ is greater than the cardinality of $A$.

(21) Let $A$ and $B$ denote non-empty sets and $f : A \rightarrow B$. Then there exists $g : B \rightarrow A$.

(22) The cardinality of $\mathbb{R}^2$ is the same as the cardinality of $\mathbb{R}$.

(23) The real numbers are uncountable.

(24) The rational numbers are countable.

(25) The positive integers are countable.

(26) The set of even integers and the set of integers have the same cardinality.
(27) For the integers, \( \mathbb{Z} \), it is true that \( \mathbb{Z} \times \mathbb{Z} \) and \( \mathbb{Z} \) have the same cardinality.

(28) The set of all infinite sequences in \{0, 1\} is countable.

(29) The set of all finite sequences in \{0, 1\} is countable.

(30) Let \( x, y \in \mathbb{R} \) and let \( n \in \mathbb{Z}^+ \). Then
\[
(x + y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}.
\]

(31) Let \( r \in \mathbb{R}, r \neq 1 \) and let \( n \in \mathbb{Z}^+ \). Then
\[
\sum_{k=0}^{n} r^k = \frac{1 - r^{n+1}}{1 - r}.
\]

(32) Let \( a_k, b_k \in \{0, 1\}, \forall k \in \mathbb{Z}^+ \). Let
\[
x = \sum_{k=1}^{\infty} \frac{a_k}{2^k} \quad \text{and} \quad y = \sum_{k=1}^{\infty} \frac{b_k}{2^k}.
\]
If \( x = y \) then \( a_k = b_k, \forall k \).

(33) The Cantor set is countable.

(34) The complement of the Cantor set in \([0, 1]\) can be written as a countable union of disjoint open intervals.

(35) Every element of the Cantor set has a unique representation of the form
\[
\sum_{k=1}^{\infty} \frac{a_k}{3^k}, \quad a_k \in \{0, 2\}, \forall k \in \mathbb{Z}^+.
\]