Example 1

Solution

<table>
<thead>
<tr>
<th></th>
<th>Nature Doc.</th>
<th>Symphony</th>
<th>Ballet</th>
<th>Opera</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sitcom</td>
<td>2</td>
<td>1</td>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>Drama</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>2</td>
</tr>
<tr>
<td>Talk Show</td>
<td>-2</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Movie</td>
<td>3</td>
<td>1</td>
<td>-1</td>
<td>1</td>
</tr>
</tbody>
</table>

Each equally likely

a) Each channel chooses randomly, how many viewers would CTV gain/lose?

\[
S = \begin{bmatrix}
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4}
\end{bmatrix}
\]

\[
T = \begin{bmatrix}
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4}
\end{bmatrix}
\]

ETV uses probabilities \( \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4} \)

CTV's pure strategy outcomes are:

\[
E_{Sitcom} = \frac{1}{4}(2) + \frac{1}{4}(1) + \frac{1}{4}(-2) + \frac{1}{4}(2)
= \frac{1}{2} + \frac{1}{4} - \frac{1}{2} + \frac{1}{2} = \frac{3}{4}
\]

\[
E_{Drama} = \frac{1}{4}(-1) + \frac{1}{4}(1) + \frac{1}{4}(-1) + \frac{1}{4}(2)
= -\frac{1}{4} + \frac{1}{4} - \frac{1}{4} + \frac{1}{2} = \frac{1}{4}
\]

\[
E_{Talk Show} = \frac{1}{4}(-2) + \frac{1}{4}(0) + \frac{1}{4}(0) + \frac{1}{4}(1)
= -\frac{1}{2} + 0 + 0 + \frac{1}{4} = -\frac{1}{4}
\]

\[
E_{Movie} = \frac{1}{4}(3) + \frac{1}{4}(1) + \frac{1}{4}(-1) + \frac{1}{4}(1)
= 3\frac{1}{4} + \frac{1}{4} - \frac{1}{4} + \frac{1}{4} = 1
\]
With CTV using mix strategy \( \text{all equally likely} \):

\[
E_{mix} = \frac{1}{4}(E_{r_1}) + \frac{1}{4}(E_{r_2}) + \frac{1}{4}(E_{r_3}) + \frac{1}{4}(E_{r_4})
\]

\[
= \frac{1}{4}(3\frac{1}{4}) + \frac{1}{4}(\frac{1}{4}) + \frac{1}{4}(-\frac{1}{4}) + \frac{1}{4}(1)
\]

\[
= 3\frac{1}{16} + \frac{1}{16} - \frac{1}{16} + \frac{1}{4} = \frac{3}{16} + \frac{1}{4} = \frac{3 + 4}{16} = \frac{7}{16}
\]

b) ETV choice is equally divided between ballet + nature documentary. What is CTVs expected gain (loss) if choose from their 4 options equally often?

ETV uses mix strategy \( T = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \)

CTVs pure strategy interior:

\[
E_{r_1} = \frac{1}{2}(2) + 0(1) + \frac{1}{2}(-2) + 0(2) = 0
\]

\[
E_{r_2} = \frac{1}{2}(-1) + 0(1) + \frac{1}{2}(-1) + 0(2) = -1
\]

\[
E_{r_3} = \frac{1}{2}(-2) + 0(0) + \frac{1}{2}(0) + 0(1) = -1
\]

\[
E_{r_4} = \frac{1}{2}(3) + 0(1) + \frac{1}{2}(-1) + 0(1) = \frac{3}{2} - \frac{1}{2} = 1
\]

With CTV mixed strategy \( \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4} \)
expected gain (loss) is

\[ E_{\text{max}} = \frac{1}{4}(0) + \frac{1}{4}(-1) + \frac{1}{4}(-1) + \frac{1}{4}(1) = -\frac{1}{4} \]

c) If CTV notices ETV is selecting a bullet half the time and a nature documentary half the time, what would CTVs best pure strategy be, and how many viewers would they gain?

From calculations above, we see that CTVs best pure strategy would be to always show a movie and they would gain 1,000 viewers.