

2.

Given  $G = (V, E)$ , edge weights  $c_e$ :

① Find optimal tour length, call it  $L$ . Let  $T = \emptyset$ .

② For  $e = 1, \dots, |E|$ :

Let  $c_e = c_e + 1$ .

Find optimal tour length,  $L_e$ .

If  $L_e \leq L$ :

edge  $e$  must be in tour.

Mark  $e$  and set  $c_e \leftarrow c_e - 1$

Let  $T \leftarrow T \cup \{e\}$ . (original value)

Else

edge  $e$  is not in tour.

Leave  $c_e$  at modified value

End if

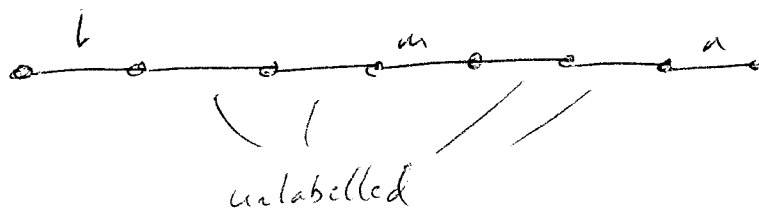
③ Optimal tour consists of edges in  $T$ .

---

The example where every  $c_e = 1$  shows that we must leave modified  $c_e$ 's at modified values if edges not in tour.

### 3. Reduce 3-SAT to Matching with Bonds:

Map clause  $C = l \vee u \vee a$  to graph component:



For every variable  $x$ , set up  $\overset{x}{\longleftarrow} \overset{\bar{x}}{\longrightarrow}$

Create bonds: every ~~literal~~ edge corresponding to a literal  $l$  is put in a bond.

Then an instance of 3-SAT with  $k$  clauses and  $v$  variables is feasible if and only if  $\exists$  matching with bonds, with  $\geq 3k + v$  edges.

4.

For  $\begin{bmatrix} 2 & x_{12} \\ x_{12} & 1 \end{bmatrix}$  to be psd, need  $-\sqrt{2} \leq x_{12} \leq \sqrt{2}$ .

So optimal value is  $\sqrt{2}$ , and optimal solution is  $x_{12} = \sqrt{2}$ .

However, this irrational number cannot be written as polynomial time, so the problem is not solvable in polynomial time.