DEFINITIONS

- subset
- proper subset
- set equality
- set union
- set intersection
- set complement
- disjoint sets
- function
  - domain of a function
  - codomain of a function
  - range of a function
  - image of a pt. (set)
  - preimage of a set
- onto function
- 1-1 function
- equal functions
- graph of a function
- composition of functions
- inverse function
- characteristic function
- group
- field
• ordered field
• upper bound of a set in \( \mathbb{R} \) (and lower bound)
• least upper bound of a set in \( \mathbb{R} \) (and glb)
• real numbers
• integers
• absolute value function
• rational number
• irrational number
• Archimedean property of \( \mathbb{R} \)
• sequence
• finite sequence
• infinite sequence
• convergent sequence
• sequence converges to a real number
• sequence converges to infinity
• Cauchy sequence
• geometric series
• real-valued function continuous at a point
• finite set
• an infinite set
• the power set of a set
• countable set
• uncountable set
• the cardinality of a finite set
• two sets have the same cardinality
• the continuum hypothesis
• the Cantor set
• the ternary representation and decimal representation of an element in \([0, 1]\).
THEOREMS, WORKSHEETS and HOMEWORKS

• Two deMorgan’s Laws (proof, see Course Notes)
• Induction Theorem (statement and proof, see Course Notes)
• A non-empty set of integers, that is bounded below, contains a least element. (proof, see Course Notes)
• $|2^S| > |S|$ (cocktail party proof, see Course Notes)
• Decimal Representation Theorem (statement, see Course Notes)
• $f(A \cup B) = f(A) \cup f(B)$ and like statements (proof, WS 3.1.2 & WS 3.3.1)
• In a group the zero element is unique and the inverse of an element is unique. (proof, WS 4.2.1)
• If $x \in \mathbb{R}$ there exists $n \in \mathbb{Z}$ such that $n \leq x < n + 1$. (proof, WS 4.4.5)
• If $f$ and $g$ are real-valued continuous functions defined on some real domain, then so too is $f + g$. (proof, WS 4.8.2)
• If $f$ is a real-valued continuous function defined on some real domain, then so too is $\lambda f$, $\forall \lambda \in \mathbb{R}$. (proof, WS 4.8.4)
• Prove that integers constitute a countably infinite set. (proof, WS 5.2.1)
• Let $\{a_k\}_{k=0}^{\infty}$ be a sequence in a field $\mathcal{F}$. Prove that

$$\sum_{k=1}^{n} (a_k - a_{k-1}) = a_n - a_0, \quad \forall n \in \mathbb{Z}^+.$$ 

Such a sum is called a **telescoping sum**. (proof, WS 6.2.4)
• $A \setminus (A \cap B) = A \cap B^c$ (proof, HW 3)
• Let $\{a_k\}_{k=0}^{\infty}$ be a convergent sequence in $\mathbb{R}$ then so is $\{\lambda x_k\}_{k=0}^{\infty}$, $\forall \lambda \in \mathbb{R}$. (proof, HW 12)
• Prove that every infinite set contains a countably infinite subset. (proof, HW 16)

TRUE/FALSE QUESTIONS

• all True/False questions from Chapters #2-7