Directions. For full credit, please submit your answer to the following problem in a LaTeX-prepared document. Class participants are encouraged to prepare solutions in a collaborative mode but to prepare their to-be-submitted write-ups individually. The consequences of sharing files, electronic or otherwise, are discussed in the course syllabus.\footnote{If the wording of this problem was discussed in detail in the classroom, the course instructor expects to see similar phrases and sentences in reading the submissions.}

Please include the problem number along with a statement of the problem in your submission. Please also include your e-mail address on your submission.

Recall that if $r \in \mathbb{R}$, $0 \leq r < 1$, then $\sum_{k=0}^{\infty} r^k = \frac{1}{1-r}$.

**Problem.** Let $\{a_k\}_{k=1}^{\infty}$ and $\{b_k\}_{k=1}^{\infty}$ be sequences in \{0, 1, \ldots, 9\}. Let $x$ and $y$ both be elements of the unit interval, $[0, 1]$, given by $x = \sum_{k=1}^{N} \frac{a_k}{10^k}$, $a_k \in \{0, 1, \ldots, 9\}$ and $y = \sum_{k=1}^{N} \frac{b_k}{10^k}$, $b_k \in \{0, 1, \ldots, 9\}$, for some positive integer, $N$. Assume that $a_k = b_k$, $k = 1, \ldots, M$, for some positive integer, $M$ where $M \leq N$. Find an estimate for $|x - y|$ that is dependent on $M$ and independent of the $a_k$’s, the $b_k$’s and $N$. 