Directions. For full credit, please submit your answer to the following problem in a \LaTeX{}-prepared document. Class participants are encouraged to prepare solutions in a collaborative mode but to prepare their to-be-submitted write-ups individually. The consequences of sharing files, electronic or otherwise, are discussed in the course syllabus.\footnote{If the wording of this problem was discussed in detail in the classroom, the course instructor expects to see similar phrases and sentences in reading the submissions.}

Please include the problem number along with a statement of the problem in your submission. Please also include your e-mail address on your submission.

Recall that $\mathbb{Z}_n$ denotes the first $n$ positive integers.

Recall the following definition.

Let $n \in \mathbb{Z}^+$. A set $S$ has \textbf{cardinality} $n$ if there exist a function $f$ such that

$$f : \mathbb{Z}_n \xrightarrow{1-1} A.$$ 

Recall also that the cardinality of the set of all functions from a set of cardinality $n$ into $\{0, 1\}$ is $2^n$.

\textbf{Problem.} Prove the following statement. Let $n \in \mathbb{Z}^+$ and let $S$ be a set containing exactly $n$ elements. Then the cardinality of the set of all subsets of $S$ is $2^n$, i.e.,

$$|2^S| = 2^n.$$ 

The phrase “a set $S$ containing exactly $n$ elements” means that the cardinality of $S$ is $n$. 