

01/16/07 Other textbooks!

Øksendal, Stochastic Differential
Equations

- pure math

Karatzas + Shreve, Brownian Motion +
Stochastic Calculus

- really technical + thorough

Gard, Stochastic Differential
Equations

- decent intermediate
mathematical treatment

Examples:

) Population Models:

$$\frac{dX}{dt} = aX(K - X)$$

Logistic equation

X = population

a = growth constant

K = carrying capacity

t = time

Uncertain: Immigration (add noise)

Environmental ~~System~~ Variations:

random fluctuations in a, K

Similar ideas for Lotka-Volterra models
& SIRS epidemics for interacting populations

2) Molecular dynamics

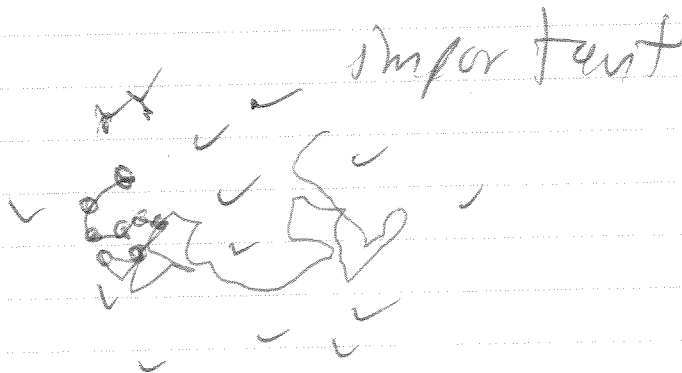
$$\overset{\text{mass matrix}}{\vec{M}} \frac{d\vec{X}}{dt} = \vec{p}$$

$$\frac{d\vec{p}}{dt} = - \vec{\nabla} V(\vec{X}) \quad \text{potential energy}$$

~~potential energy~~

\vec{X} = positions of atoms
 \vec{p} = momenta
} super vectors

~~Large~~ Large biomolecules: water molecules



T. Schlick, Molecular Modeling

Coarse-grain the molecule or solute, and the effects of water on microscale.

On large enough scales (microns), water's main effect might be hydrodynamic

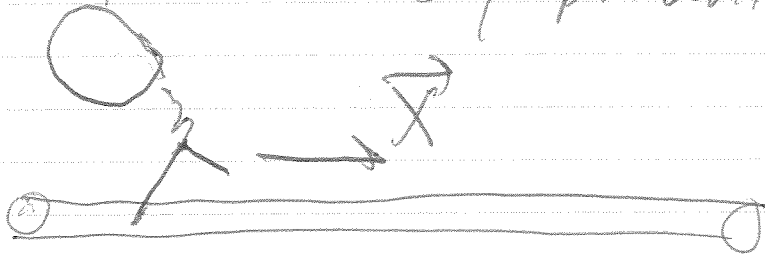
$$\overline{M} \frac{d\vec{X}}{dt} = \vec{P}$$

friction matrix

$$\frac{d\vec{P}}{dt} = -\vec{\nabla} V(\vec{X}) - \underbrace{\Gamma(\vec{X}) \vec{P}}_{\substack{\text{dissipation} \\ \text{from coupling to} \\ \text{fluid}}}$$

$$+ \underbrace{F_T(\vec{X}, t)}_{\text{stochastic thermal forcing}}$$

3) Molecular motors (Brownian motors)



$$\gamma \frac{d\vec{x}}{dt} = -\vec{\nabla} V(\vec{x}(t), f(t)) + \gamma(t)$$

viscous coefficient + thermal noise

\vec{x} : molecular motor position

$f(t), \gamma(t)$: random inputs that modulate the potential