Math 1020

Exam 4 - Possible Short Answer Questions

Exam 4 covers sections 8.1, 8.2, 8.3, 8.6, 8.7, 8.8, 8.9, 8.10, and 2.8

Multiple choice questions can be conceptual (like many of the T/F questions) or involve some of the easier calculations found in this question list.

For all interval of convergence questions - do not worry about the endpoints of the intervals.

The True/False questions from the text are

8.1# 113-116 8.8 # 59-62
8.2# 99-102 2.8 # 49-52

1. Given the sequence \( \frac{1}{2}, -\frac{3}{4}, \frac{9}{8}, -\frac{27}{16}, \ldots \)
   
   (a) Write an expression for the \( n^{th} \) term of the sequence.
   
   (b) Does the sequence, with the \( n^{th} \) term you found in a), converge or diverge?

2. (a) Write out the first five terms, \( a_1, ..., a_5 \) of the sequence \( \left\{ \frac{n \cos(n\pi)}{3n+2} \right\} \). Be sure to evaluate all trig functions and reduce fractions in your answer.
   
   (b) Does the following sequence with the given \( n \)th term converge or diverge? If the sequence converges, find its limit.
   
   \[ b_n = \frac{2n + 1}{n + 3} \]

3. Write down the iterative formula for Newton’s Method when approximating the zeros of \( f(x) = e^x + x \).

4. Given \( f(x) = x^2 + 3x + 3 \), use Newton’s Method with initial approximation \( x_1 = 1 \) to find \( x_3 \).

5. Determine whether the series converges or diverges. If it converges, find the sum.
   
   \[ 5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \cdots \]

6. Determine whether the series converges or diverges. If it converges, find the sum.
   
   \[ \sum_{n=0}^{\infty} 4^{n+1} \frac{1}{5^{n}} \]

7. What is the sum of the following series? \[ \sum_{n=0}^{\infty} \frac{1}{2^n} - \frac{1}{3^n} \]

8. What is the sum of the following series? \[ \sum_{n=1}^{\infty} \frac{1}{n+1} - \frac{1}{n+2} \]

9. Determine if the following series converges or diverges. Be sure to state the test you used to determine convergence or divergence. (\( n^{th} \) term, geometric, telescoping, integral, root, or ratio.)
   
   \[ \sum_{n=1}^{\infty} \frac{n + 10}{10n + 1} \]
10. Find the interval and radius of convergence for the power series given below.

\[ \sum_{n=1}^{\infty} \frac{(3x - 2)^n}{n \cdot 4^{n+1}} \]

11. Given \( f(x) = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x-1)^{n+1}}{n+1} \),
   (a) find the interval of convergence for \( f(x) \). interval.
   (b) find \( f'(x) \) and the interval of convergence for \( f'(x) \). interval.
   (c) find \( \int f(x) \, dx \) and the interval of convergence for \( \int f(x) \, dx \).

12. Find the geometric power series \( f(x) = \frac{3}{2x-1} \) centered at \( c = 0 \) and identify the interval of convergence.

13. Find the geometric power series \( f(x) = \frac{1}{2-x} \) centered at \( c = 5 \) and identify the interval of convergence.

14. Given \( f(x) = \frac{-1}{(x+3)^2} = \frac{d}{dx} \left[ \frac{1}{x+3} \right] \), find a power series for \( f(x) \) centered at 0.

15. Given \( f(x) = \ln(x+2) = \int \frac{1}{x+2} \, dx \), find a power series for \( f(x) \) centered at 0.

16. Given the power series for \( e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \), find the Maclaurin series for the function:

\[ f(x) = e^{x^2} \]

17. (a) Given \( f(x) = \frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n \) for \(-1 < x < 1\), what is the power series for \( f(4x^2) = \frac{1}{1+4x^2} \)?
   (b) Using the result above and the equation \( \frac{-8x}{(1+4x^2)^2} = \frac{d}{dx} \left( \frac{1}{1+4x^2} \right) \), find a power series for \( \frac{-8x}{(1+4x^2)^2} \) centered at 0. What is the interval of convergence for the power series?

18. (a) Find the 4th degree Taylor polynomial, \( P_4 \), for \( f(x) = \frac{1}{x} \) centered at \( c = 1 \).
   (b) Write out the Taylor Series for \( f(x) = \frac{1}{x} \) centered at \( c = 1 \). Be sure to find the pattern and write your answer in summation notation.
   (c) Find the upper bound on the error if you use \( P_4 \) found in (a) to approximate \( f(x) \) on the interval \([1, 1.2]\).

Exam # 4 Formula Sheet

Taylor Remainder and Inequality on an interval

\[ f(x) = P_n(x) + R_n(x) \]

\[ |R_n(x)| = \frac{|f^{(n+1)}(z)|}{(n+1)!}|x-c|^{n+1} \leq \frac{M}{(n+1)!}d^{n+1} \]

where \( M \) is the maximum value of \(|f^{(n+1)}|\) on the interval and \( d \) is the length of the interval.