Write your Section Number

Name ________________________________

Instructions.
Please work all four problems.

Please show all work clearly and in reasonable detail. Answers without appropriate supporting work or requested explanations may not receive full credit.

Use of books, notes, or calculators is not permitted.

1. (a) Find the critical point for the function

\[ f(x, y) = x^2 - xy + y^2 + 9x - 6y + 10, \]

and show that it is a local minimum.

(b) Which one of the following represents the surface of part (a)?

(c) Write an equation for the tangent plane to the surface

\[ z = x^2 - xy + y^2 + 9x - 6y + 10, \text{ at } (-1, 1, -2). \]
2. The temperature at a point \((x, y)\) on a flat metal plate which occupies the region
\[ R = \{(x, y) \mid 0 \leq x \leq 10, \ 0 \leq y \leq 2\pi\} \] is given by
\[ T(x, y) = 60 + 40e^{-x} \cos y, \]
where \(T\) is measured in °C and \(x, y\) in centimeters.

(a) Show that \(T_y(x, 0) = 0\). This means that the edge \(y = 0\) is insulated. Is the edge \(y = 2\pi\) insulated? (Evaluate \(T_y(x, 2\pi)\).)

(b) Find the rate of change of temperature with respect to distance at the origin in the direction of the point \((4, 3)\).

(c) In what direction should an ant at the point \(\left(\frac{1}{2}, \frac{\pi}{2}\right)\) on the plate, move in order to cool off as rapidly as possible?

(d) Determine whether or not the temperature distribution is in a state of equilibrium so that \(T\) satisfies Laplace’s equation
\[ u_{xx} + u_{yy} = 0. \]

3. Consider the function
\[ V = f \left( \frac{x}{z}, \frac{y}{z} \right). \]
Use the chain rule to show that
\[ xV_x + yV_y + zV_z = 0. \]

4. The variables \(x\) and \(y\) are functions of \(r\) and \(s\) determined by
\[ x^2 - y^2 + 2r = 0 \text{ and } xy - s = 0. \]
Show that
\[ \frac{\partial x}{\partial r} = -\frac{x}{x^2 + y^2} \text{ and find } \frac{\partial x}{\partial s}. \]