This assignment is in two parts. The answers to questions in Part I are generally in the books. It is advisable to make every effort to solve the problem before consulting the answer. Page numbers are to the Text, Introduction to Linear Algebra, 5th edition, by Johnson, Riess, and Arnold.

Part I

Section 3.2

1. (a) Text: p.175, #29.
   (b) Text: p.175, #32

Section 3.3

2. (a) Text: p.187, #25
   (b) Text: p.188, #43.

Sections 3.4-3.5

3. Text: p.200, #11

4. (a) Text: p.201, #29
   (b) Text: p.213, #29

Part II

5. **Traces.** If $C$ is an $(n \times n)$ matrix, then the trace of $C$ is $\text{tr}(C) = \sum_{i=1}^{n} c_{ii}$, which is the sum of its diagonal elements.
   (a) If $A$ is $(m \times n)$ and $B$ is $(n \times m)$, use Definition 8 of chapter 1 to show that $\text{tr}(AB) = \text{tr}(BA)$.
   (b) Verify the conclusion of part (a) when

   \[ A = \begin{bmatrix} -3 & 0 \\ 2 & -1 \\ -2 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 3 & 0 \end{bmatrix}. \]

6. Given that $A$ is a $(2 \times 2)$ matrix such that $A^2 + 3A - I = O$, where $I$ is the identity and $O$ is the zero matrix. Suppose we know that

   \[ Au = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad \text{where} \ u = \begin{bmatrix} 1 \\ 3 \end{bmatrix}. \]

   (a) Find $A^2u$ and $A^3u$.
   (b) Show that $A$ is nonsingular. (Hint: Is there a nonzero vector $x$ such that $Ax = 0$?) Find $A^{-1}u$.
   (c) Using the fact that $A^2 = I - 3A$, we can find scalars $a_k$ and $b_k$ such that $A^k = a_kA + b_kI$. Find these scalars for $k = 2, 3, \text{and} 4$. 

(over)
7. (MAPLE) (a) Let $S$ denote the subspace of $\mathbb{R}^5$ spanned by the set

$$\left\{ \begin{bmatrix} 2 \\ 3 \\ 1 \\ 6 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 1 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ 8 \\ 2 \\ 2 \\ 1 \end{bmatrix} \right\}.$$ 

Determine if \[
\begin{bmatrix} 6 \\ 2 \\ 1 \\ 11 \\ 13 \end{bmatrix}
\]
is in $S$.

(b) Determine if the following set is a basis for $\mathbb{R}^5$:

$$\left\{ \begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 2 \\ 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \end{bmatrix} \right\}.$$ 

8. Let

$$W = \left\{ y = \begin{bmatrix} x_1 - x_2 + x_3 \\ -x_1 + x_2 - x_3 \\ x_1 - x_2 + x_3 \end{bmatrix} : x_1, x_2, x_3 \text{ real} \right\}.$$ 

Exhibit a $(3 \times 3)$ matrix $A$ such that $W = \mathcal{R}(A)$. Conclude that $W$ is a subspace of $\mathbb{R}^3$. What is $\dim(W)$? Find a basis for $W$. 

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