This assignment is in two parts. The answers to questions in Part I are generally in the book. It is advisable to make every effort to solve the problem before consulting the answer.

Part I

Section 14.2

1. p.1031, #63

Section 14.3

2. p.1039, #3

3. (a) p.1039, #5.
   (b) p.1040, #29. Note: There is a MAPLE part as well.
   (c) Compare this problem with Problem 2 (a) of Assignment #4: p.1017, #41.

Section 14.4

4. p.1049, #15.

Part II

Section 14.2

5. **Center of mass of a wire.** Consider a wire with density function \( \rho(x, y, z) \) in the shape of a space curve \( \gamma : r(t) = x(t)i + y(t)j + z(t)k, \ a \leq t \leq b \). The center of mass of the wire, of mass \( M \), is at the point \((\bar{x}, \bar{y}, \bar{z})\), where

\[
\bar{x} = \frac{1}{M} \int_C x\rho(x, y, z)ds, \quad \bar{y} = \frac{1}{M} \int_C y\rho(x, y, z)ds, \quad \bar{z} = \frac{1}{M} \int_C z\rho(x, y, z)ds.
\]

Find the center of mass of a wire when \( r(t) = \frac{1}{\sqrt{2}}(\cos t i + \sin t j + tk), \ 0 \leq t \leq 6\pi \), and \( \rho = k(1 + z), \ k \) constant. **Hint:** See Example 5 on p. 1023.

Section 14.3

6. Find the work done by the force field

\[
\mathbf{F}(x,y) = \frac{2}{y^2}e^{2x/y}(yi - xj)
\]

in moving an object from \( P(-3,2) \) to \( Q(1,4) \).
Green’s Theorem in polar coordinates. Refer to problems 6 and 7 of Assignment #4.

(a) If in Cartesian coordinates $dr = dx\,i + dy\,j$, show that in polar coordinates where $x = r\cos\theta, y = r\sin\theta$, $dr = dr_e_r + r\,d\theta e_{\theta}$. 

(b) Let $\mathbf{F}(r, \theta) = M(r, \theta)e_r + N(r, \theta)e_{\theta}$, show that

$$\int_C \mathbf{F} \cdot dr = \int_C M\,dr + rN\,d\theta.$$ 

Hence show that another alternative form of Green’s theorem is

$$\int_C M\,dr + rN\,d\theta = \iint_R \left( \frac{\partial}{\partial r}(rN) - \frac{\partial M}{\partial \theta} \right) dr\,d\theta$$

(c) Use the polar form to verify both sides of Green’s Theorem when

$$\mathbf{F} = (x^2 + y^2)^{1/2}(-yi + xj),$$

and $C$ is the circle $x^2 + y^2 = a^2$.

Note: The vector form may be written

$$\int_C \mathbf{F} \cdot dr = \iint_R \left( \frac{1}{r} \frac{\partial}{\partial r}(rN) \right) r\,dr\,d\theta = \iint_R \text{curl} \, \mathbf{F} \cdot kdA,$$

where the quantity $\frac{1}{r} \frac{\partial}{\partial r}(rN) - \frac{1}{r} \frac{\partial M}{\partial \theta} = \text{curl} \, \mathbf{F}$ in polar coordinates.