Separation of Variables

Step A. Find many solutions to

\[ u_t = u_{xx} \]

\[ u(0, t) = 0 = u(L, t) \]

- A1. Look for a solution of the form \( u(x, t) = X(x)T(t) \), plug into DE, and separate variables

- A2. The \( X \) problem is

\[ X'' + \sigma X = 0 \]

\[ X(0) = 0 = X(L) \]

Solve this two-point boundary value problem.
• A3. The $T$ problem is $T' + \sigma T = 0$. Solve this first-order linear ODE.

• A4. Put the solutions of A2 and A3 together: $u_n(x, t) = X_n(x)T_n(t)$

Step B. Take linear combinations of the solutions from A to solve $u(x, 0) = f(x)$. Get a Fourier series problem.