Separation of Variables

Step A. Find many solutions to

$$u_t = u_{xx}$$

$$u(0,t) = 0 = u(L,t)$$

- A1. Look for a solution of the form $u(x,t) = X(x)T(t)$, plug into DE, and separate variables

- A2. The X problem is

$$X'' + \sigma X = 0$$

$$X(0) = 0 = X(L)$$

Solve this two-point boundary value problem.

- A3. The T problem is $T' + \sigma T = 0$. Solve this first-order linear ODE.

- A4. Put the solutions of A2 and A3 together: $u_n(x,t) = X_n(x)T_n(t)$

Step B. Take linear combinations of the solutions from A to solve $u(x,0) = f(x)$. Get a Fourier series problem.