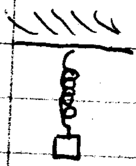
§ 3.8 Mechanical Vibration

2-18-04



$$m\ddot{u} + \gamma\dot{u} + ku = F(t)$$

m ← mass, γ ← damping const, k ← spring const., $F(t)$ ← external force

$$u(0) = u_0$$
$$\dot{u}(0) = \dot{u}_0$$

undamped free vibrations (harmonic oscillator)

$$m\ddot{u} + ku = 0 \qquad \text{Try } u = e^{rt}$$

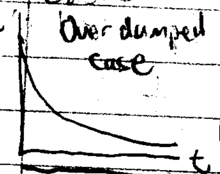
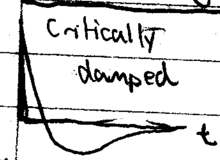
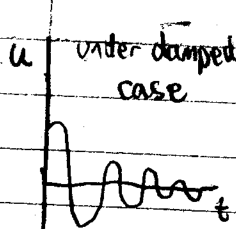
$$r = \pm\underbrace{\sqrt{\tfrac{k}{m}}}_{w_0}$$

$$u = A\cos w_0 t + B\sin w_0 t$$
$$= \underset{\text{Amplitude}}{R}\cos(w_0 t + \underset{\text{phase}}{\delta})$$

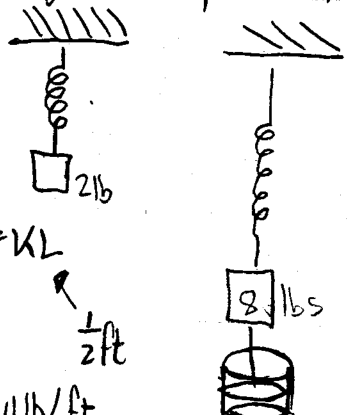
Damped free vibrations

$$m\ddot{u} + \gamma\dot{u} + ku = 0 \qquad \text{Try } u = e^{rt}$$
$$mr^2 + \gamma r + k = 0$$
$$r = \frac{-\gamma \pm \sqrt{\gamma^2 - 4mk}}{2m}$$

Cases:

	$\gamma^2 - 4km$	roots (r_1, r_2)	general solutions to D.E.
 Over damped case	+	2 different real roots; $r_1, r_2 < 0$	$u = Ae^{r_1 t} + Be^{r_2 t}$; note r_1, r_2 are negative - damping exponentials
 Critically damped	0	1 negative real root $r = -\gamma/2m$	$u = Ae^{-\gamma/2m\,t} + Bte^{-\gamma/2m\,t}$ $= (A + Bt)e^{-\gamma/2m\,t}$
 under damped case	-	$r = \lambda \pm i\mu$; $\lambda = -\gamma/2m$; $\mu = \frac{\sqrt{4mk - \gamma^2}}{2m}$	$u = Ae^{\lambda t}\cos\mu t + Be^{\lambda t}\sin\mu t$ $= e^{\lambda t}(A\cos\mu t + B\sin\mu t)$

Ex A 2 lb weight stretches spring 6 inches. A dashpot applies a force of 1 lb at velocity 1 ft/sec. If an 8 lb weight is attached to the spring and released from a position 1/2 ft below the equilibrium, find the motion.



$F_s = KL$

2lb ↗ $\frac{1}{2}$ft

$K = 4 \text{ lb/ft}$

$F_d = -\gamma \frac{du}{dt}$

$1\text{lb} = -\gamma \; 1 \text{ ft/sec}$

$\gamma = 1 \text{ lbs-sec/ft}$

$F_g = mg$ ← 32.2 ft/sec^2

8lb

$m = \frac{1}{4} \frac{\text{lb sec}^2}{\text{ft}} = \frac{1}{4}$ slug

$m\ddot{u} + \gamma\dot{u} + ku = 0$

$\frac{1}{4}\ddot{u} + 1\dot{u} + 4u = 0$

$u(0) = \frac{1}{2}$

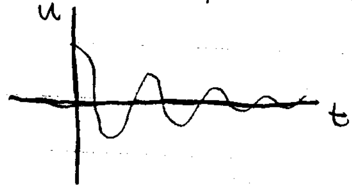
$\dot{u}(0) = 0$

Try $u = e^{rt}$

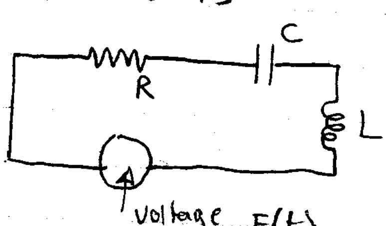
$\frac{1}{4}r^2 + r + 4 = 0$

$r = \dfrac{-1 \pm \sqrt{1 - 4 \cdot \frac{1}{4} \cdot 4}}{(2\cdot\frac{1}{4})}$

underdamped



Electrical circuits



"RLC circuits"

R = resistance

L = inductance

C = capacitance

Kirchhoff law: sum of voltage drop around a closed circuit is 0

Voltage drop across resistor = IR current $= \frac{dQ}{dt}$ ← charge

Voltage drop across capacitor $= \dfrac{Q}{C}$

" " " " " inductor $= L\dfrac{dI}{dt} = L\dfrac{d^2Q}{dt^2}$

$$L\frac{dQ^2}{dt} + R\frac{dQ}{dt} + \frac{Q}{C} = E(t)$$

correspondance with spring mass system

Circuit	spring mass system
L	m
R	γ
$1/c$	K
	$m\ddot{u} + \gamma\dot{u} + ku = F(u)$