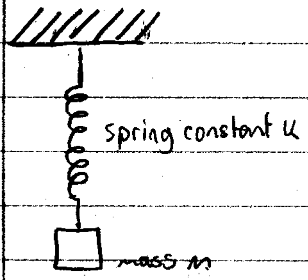
2-13-04

3.8 Mechanical Vibration



damping constant γ

u = displacement from equilibrium

$$m\ddot{u} + \gamma\dot{u} + ku = F(t) \leftarrow \text{applied force}$$

Free vibrations (harmonic oscillator)

undamped free vibrations

$\gamma = 0 \quad F(t) = 0 \quad m\ddot{u} + ku = 0$

Try $u = e^{rt} \quad mr^2 + k = 0 \quad r = \sqrt{\frac{-k}{m}} = \sqrt{\frac{k}{m}}$

$$u = c_1 \cos\sqrt{\tfrac{k}{m}}\,t + C_2 \sin\sqrt{\tfrac{k}{m}}\,t$$

$$u_+ = e^{\sqrt{\frac{k}{m}}it} = \cos\sqrt{\tfrac{k}{m}}\,t + i\sin\sqrt{\tfrac{k}{m}}\,t$$

$$u_- = e^{-\sqrt{\frac{k}{m}}it} = \ldots$$

$\omega_0 = \sqrt{\frac{k}{m}}$ natural (circular / angular) frequency

$$\boxed{u = C_1\cos\omega_0 t + C_2\sin\omega_0 t}$$

$u = A\cos\omega_0 t + B\sin\omega_0 t$

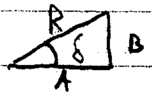
commonly written $u = R\cos(\omega_0 t + \delta)$

use ∡ addition formula

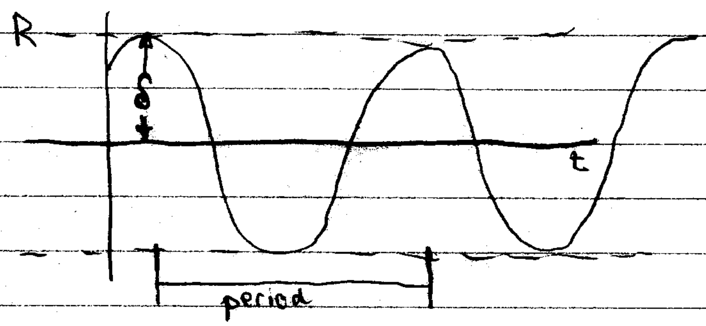
$$= R(\cos\omega_0 t\cos\delta + \sin\omega_0 t\sin\delta$$

$A = R\cos\delta \qquad B = R\sin\delta$

$\boxed{\text{amplitude}} \Rightarrow R = \sqrt{A^2+B^2}$



$\tan\delta = \frac{B}{A} \qquad \delta = \boxed{\text{angle phase}}$



period

$\omega_0 t : 0 \longrightarrow 2\pi$

$t : 0 \longrightarrow \frac{2\pi}{\omega_0}$

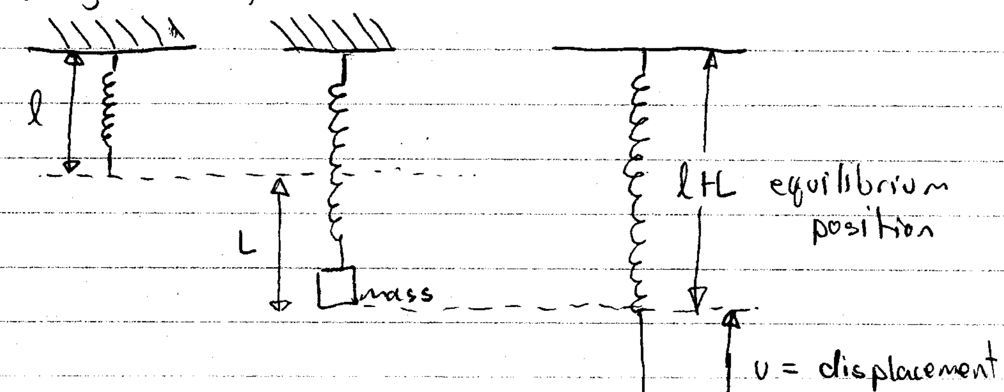
period also written

period $= 2\pi\sqrt{\frac{m}{k}}$

frequency $= \frac{1}{\text{period}} = \frac{\omega_0}{2\pi}$

§ 3.8 Mechanical Vibrations

spring mass systems



To analize motion use: $F = ma$

What are forces on mass?

$$F = \underset{\text{gravity}}{F_g} + \underset{\text{spring}}{F_s} + \underset{\text{damping}}{F_d} + \underset{\text{applied}}{F_a}$$

1) F_g due to weight $F_g = mg$

2) F_s (spring force $\propto$ elongation or compression of spring } hooks Law

$$F_s = -K(L+u)$$

3) F_D damping force $\propto$ speed

$$F_D = -\gamma \frac{du}{dt}$$

$F = ma$ is

$$\cancel{mg} - K(\cancel{L}+u) - \gamma \frac{du}{dt} + F(t) = m\frac{d^2u}{dt^2}$$

$$\boxed{m\frac{d^2u}{dt^2} + \gamma \frac{du}{dt} + ku = F(t)}$$