~~Other cases: $b^2-4ac=0$ → double root r~~
~~get $y=e^{rt}$~~
~~$b^2-4ac<0$ complex roots~~

Real equal roots $b^2-4ac=0$

Ex) $y''+2y'+y=0$ try $y=e^{rt}$

$$r^2\cancel{e^{rt}}+2r\cancel{e^{rt}}+\cancel{e^{rt}}=0$$

$$(r+1)^2 = r^2+2r+1=0 \qquad r=-1$$

$y=e^{-t}$ is the only solution we get from this procedure

$y=c_1e^{-t}$

We want anothe solution

idea: look for a second solution in the form $y=v(t)e^{-t}$, try to find v.

$$y'=v'e^{-t}-ve^{-t}$$

plug in $\quad y''=v''e^{-t}-2v'e^{-t}+ve^{-t}$

$$v''\cancel{e^{-t}}-2v'\cancel{e^{-t}}+v\cancel{e^{-t}}+2(v'\cancel{e^{-t}}-v\cancel{e^{-t}})+v\cancel{e^{-t}}=0$$

$$v''+v'\underbrace{(-2+2)}_{0}+v\underbrace{(1-2+1)}_{0}=0$$

$$v''=0$$

$$v(t)=at+b$$

get second solution to ODE

$$y=(at+b)e^{-t}$$

general solution to ODE

$$y=c_1e^{-t}+c_2(at+b)e^{-t}=e^{-t}(c_1+c_2b)+te^{-t}(c_2a)$$
$$=a_1e^{-t}+a_2te^{-t}$$

$ay'' + by' + cy = 0$ 2-4-04

$ar^2 + br + c = 0$ $\qquad r = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$b^2 - 4ac$	roots	general solution
positive	r_1, r_2 real	$c_1 e^{r_1 t} + c_2 e^{r_2 t}$
negative	$r = \lambda \pm i\mu$	$c_1 e^{\lambda} \cos \mu t + c_2 e^{\lambda t} \sin \mu t$
zero	double root	$c_1 e^{rt} + c_2 t e^{rt}$

Ex $\quad y'' - 6y' + 9y = 0$ $\qquad$ Try $y = e^{rt}$

$(r^2 - 6r + 9)\cancel{e^{rt}} = 0$

$(r-3)^2 = 0 \Rightarrow r = 3$

$y = c_1 e^{3t} + c_2 t e^{3t}$

Say initial conditions are

$y(0) = 1$

$y'(0) = 1$

$1 = y(0) = c_1$

$y' = 3c_1 e^{3t} + c_2 e^{3t} + 3c_2 t e^{3t}$

$1 = y'(0) = 3c_1 + c_2 \Rightarrow c_2 = -2$

$y = e^{3t} - 2te^{3t}$

Ex $\quad y'' + 7y' + 9y = 0$ $\qquad$ Try $y = e^{rt}$

$r^2 + 7r + 9 = 0$ $\qquad r = \dfrac{-7 \pm \sqrt{13}}{2}$

$y = c_1 e^{\frac{-7+\sqrt{13}}{2} t} + c_2 e^{\frac{-7-\sqrt{13}}{2} t}$

Ex $\quad y'' + 5y' + 9y = 0$

$r^2 + 5r + 9 = 0$

$r = \dfrac{-5 \pm \sqrt{-11}}{2} = \dfrac{-5}{2} \pm \dfrac{\sqrt{11}}{2} i$

$y = c_1 e^{-5/2 t} \cos \frac{\sqrt{11}}{2} t + c_2 e^{-5/2 t} \sin \frac{\sqrt{11}}{2} t$