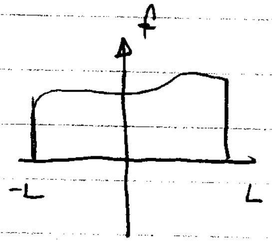
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L} \right)$$

use $\displaystyle\int_{-L}^{L} \cos\frac{n\pi x}{L} \cos\frac{m\pi x}{L}\, dx = \begin{cases} 0 & m \neq n \\ L & m = n \neq 0 \\ 2L & m = n = 0 \end{cases}$



$$\int_{-L}^{L} \cos\frac{n\pi x}{L} \sin\frac{m\pi x}{L}\, dx = 0$$

$$\int_{-L}^{L} \sin\frac{n\pi x}{L} \sin\frac{m\pi x}{L}\, dx = \begin{cases} 0 & m \neq n \\ L & m = n \end{cases}$$

$$\int_{-L}^{L} \cos\frac{2\pi x}{L} f(x) = \int_{-L}^{L} \cos\frac{2\pi x}{L} \frac{a_0}{2} + \int_{-L}^{L} \cos\frac{2\pi x}{L}\, a_1 \cos\frac{\pi x}{L} + \int_{-L}^{L} \cos\frac{2\pi x}{L}\, a_2 \cos\frac{2\pi x}{L}$$

$$+ \int_{-L}^{L} \cos\frac{2\pi x}{L}\, a_3 \cos\frac{3\pi x}{L}\, dx + \ldots$$

$$+ \int \cos\frac{2\pi x}{L}\, b_1 \sin\frac{\pi x}{L}\, dx + \int \cos\frac{2\pi x}{L}\, b_2 \sin\frac{\pi 2x}{L}\, dx$$

$$= a_2 L$$

$$a_2 = \frac{1}{L} \int_{-L}^{L} f(x) \cos\frac{2\pi x}{L}\, dx$$

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos\frac{n\pi x}{L}\, dx \qquad n \neq 0$$

and

$$b_2 = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{2\pi x}{L}\,dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$

$$\boxed{\begin{aligned} f(x) &= \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi x}{L} + b_n\sin\frac{n\pi x}{L}\right) \\ a_n &= \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx \\ b_n &= \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx \end{aligned}}$$

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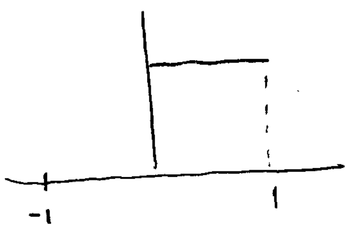
Ex

$$f(x) = \begin{cases} 1 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$



$$f(x) = \frac{a_0}{2} + \sum \left(a_n\cos n\pi x + b_n \sin n\pi x\right)$$

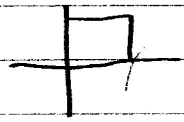
$$a_n = \frac{1}{1}\int_0^1 1\cos n\pi x\,dx \left(= \int_0^1 + \cancel{\int_{-1}^{0} 0} \right)$$

$$= \frac{1}{n\pi}\sin n\pi x \Big|_0^1 = \frac{1}{n\pi}(0-0) = 0 \quad \left(\text{for } n \neq 0\right)$$

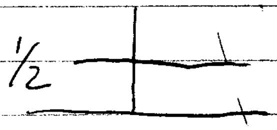
$$a_0 = \int_0^1 1\cos \xrightarrow{} \cancel{0\pi x}\ 1\ dx = 1$$

$$b_n = \frac{1}{1}\int_0^1 1\sin n\pi x\,dx = -\frac{\cos n\pi x}{n\pi}\Big|_0^1$$

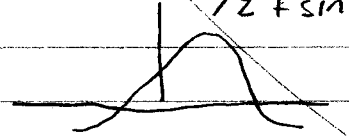
$$= -\frac{1}{n\pi}\begin{cases} 1-1=0 & n \text{ even} \\ -1-1=-2 & n \text{ odd} \end{cases} \qquad \begin{cases} 0 & n \text{ even} \\ 2/n\pi & n \text{ odd} \end{cases}$$

For 

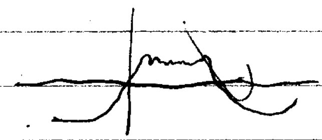
$$f(x) = \begin{cases} 1 & 0 \le x \le 1 \\ 0 & -1 < x < 0 \end{cases} = \tfrac{1}{2} + \tfrac{2}{\pi}\sin \pi x + \tfrac{2}{3\pi}\sin 3\pi x + \tfrac{2}{5\pi}\sin 5\pi x + \dots$$

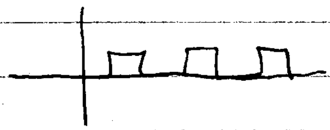


1/2 + sin πx

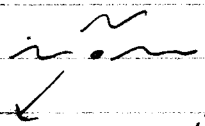


$\tfrac{1}{2} + \tfrac{2}{\pi}\sin \pi x + \tfrac{2}{3\pi}\sin 3\pi x$





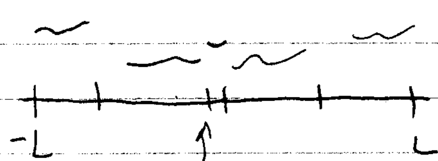
We start with a function f defined on [-L, L]. Outside that internal, the fourier series converges to the periodic extension of f



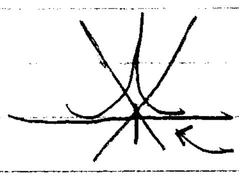
10.3 The Fourier theorm

If f and f' and piece wise continuous on [-L, L] and defined outside [-L, L] so that it is periodic with perid 2L. Then the fourier series ☆ converges to fo at points where f is continuous, and where f jumps, the series converges to $\frac{f(x^+) + f(x^-)}{2}$

f is piecewise continuous if



f is continuous on each subinterval and has a finite limit at the end points



← not piece wise contin

$f(x^+) = \lim_{\substack{y \to x \\ y > x}} f(x)$ $\qquad$ $f(x^-) = \lim_{\substack{y \to x \\ y < x}} f(x)$

$f(x^+)$ - - - -

$f(x^-)$ - - - -